Maneuvering Capabilities of High Payload Fraction Airframes

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The objective of this work is to document weight dependent trends in aircraft maneuverability and agility. High payload fraction airframes designed and certified to seemingly modest limiting gee levels may display surprisingly high levels of maneuverability and agility when flown at light weight. This work also provides a concise history of civilian structural certification loads, develops relevant performance equations consistent with US military standards. It concludes with a showcase of the turning performance of a typical narrow body transport aircraft (reminiscent of an Airbus A320) flown at a variety of weights and demonstrate that has competitive maneuverability with a famous third-generation fighter.

I. Introduction

VISITOR'S to Europe's major airshows witness thrilling flight displays performed by apparently non-aerobatic aircraft. On July 18, 2018, Lockheed-Martin test pilot Wayne Roberts flew a LM-100J (a 14 CFR § 25 certified variant of the C-130J-30 military transport) through a complete inverted loop; see FIGURE 1. [1][2] Less breathtaking, but a similarly impressive showcase of energy maneuverability was seen with the RAI Leonardo C-27J display at the RAF Fairford Royal International Air Tattoo. While I did not personally witness the LM-100 loop, I took the photograph of near inverted flight of the C-27J at the 2017 Tattoo; see FIGURE 2.

FIGURE 1 – LM-100 flying an inverted loop at Farnborough [1]

The uninformed may wonder if these aircraft are specially prepared for such flights; but, all evidence points opposite. These are otherwise "stock" transport aircraft operated outside of "handbook" procedures but clearly within their aerodynamic and structural limits and flown at weights far lighter than seen in typical operations.

FIGURE 2 – C-27J near inverted at the 2017 RIAT [author's collection]

This paper prompts a discussion relating to factors which govern the maneuverability of high-payload fraction aircraft flown far below their certification weight.

We document the many subtle interrelations between wing size, thrust levels, stall characteristics and the "*V-n*" diagram to see how they impact instantaneous and sustained turn capabilities of aircraft.

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II. Structural Certification Basis

The basic idea to proof-test aircraft structures to some multiple of its maximum certified flight weight goes back to the earliest days of Federal Regulation, predating even the Code of Federal Regulations. The U.S. Department of Commerce Aeronautics Bulletin 7-A, from the early 1930's, stipulates that all Federally Certified aircraft must demonstrate that its structure "shall be of sufficient strength to withstand the stresses imposed upon them by (designated) flying conditions and load factors." [3] From the outset, proof loads have been defined in terms of maximum expected flight loads, including inertial relief credit from "dead weight." [3]

A. History of the *V-n* Diagram

The modern concept of a load-factor vs flight speed design envelope, the *V-n* diagram, emerges in the initial edition of the Code of Federal Regulations, published 1938. 14 CFR § 04 "Airplane Airworthiness" (1938) [4] is the direct precursor to the modern 14 CFR § 23 [5] and 14 CFR § 25 [6] regulatory framework. For example:

- 14 CFR § 04.119 (1938) defines the Load Factor, *n*, as "the ratio of a load to the design weight. When the load in question represents the net external load acting on the airplane in a given direction, n represents the acceleration factor in that direction." [7]
- 14 CFR § 04.120 (1938) defines the Limit Load as the "load (or load factor, or pressure) which it is assumed or known may be safely experienced but will not be exceeded in operation" [8]

and

- 14 CFR § 04.121 (1938) defines the Factor-of-Safety as "a factor by which the limit loads are multiplied for various design purposes." [9]

Certification requires demonstration that structure withstands a strength test, that is a static test where the structure survives application of the limit load multiplied by the factor-of-safety.

In general, the structrual airworthiness of an airplane is based on the the airspeeds and accelerations (from maneuvering or gusts) which can be developed in controlled flight. [10] Regulations sketch out corner points of the basic symmetrical (i.e. zero-sideslip angle) flight envelope with flaps retracted. [11] These corner points were based on: 1) the maximum level flight speed, *VL*, which is the indicated airspeed in level flight at *MTOW* at full design power. [12], 2) the design gliding speed, *Vg*, which is the maximum indicated airspeed used to determine the flight loads; *Vg>VL* [13], 3) the design stalling speed, Vs, the indicated airspeed in unaccelerated flight based on *CLmax* at *MTOW* [14] and 4) the maximum vertical speed, *Vm*, a fictitious value of indicated airspeed computed for unaccelerated gliding flight (i.e. zero thrust); *Vm>Vg>VL* [15] and 5) the design maneuvering speed, *Vp*, the indicated airspeed at which maximum operation of control surfaces was assumed. [16]

This early CFR separates wing primary structure design guidelines from empennage and control surface design. Wing structure must be sized to maximum-takeoff-weight (*MTOW*) and design-load-factor (*nZcert*) standards; empennage and control surfaces will be designed to loads associated with full-deflection associated with flight at the design maneuvering speed. [17][18][19][20]

The corner points of these 1938 vintage structural design standards are recognizable to today's engineer. Regulations stipulate that all design cases (for wing, empennage and control surfaces) shall not fail when stressed to the associated Limit Load multiplied by a 1.5 Factor-of-Safety. [18]

These regulations define the gust load factor in terms of a peculiar function:

$$\Delta n_{gust} = \frac{K\, U\, V\, m}{575\,(\frac{W}{S})} \quad (1)$$

where $K = min(0.5(W/S)^{1/4}, 1)$, *U* is the gust velocity in ft/sec, and *V* is the indicated airspeed associated with the flight conditions and *m* is the slope of $dCL/d\alpha$ per radian.

Condition I represents operation at the maximum level flight speed (*VL*) with a *nZcert* not less than 2.5-gee's or the greater of the 1-gee plus a gust load factor based upon a +30 ft/sec gust at the *VL* speed and 1-gee plus a *Δn* value interpolated from a chart based upon gross weight (i.e. *MTOW*) and the power loading. The greater the power loading (i.e. the weight per installed horsepower), the lower the design load factor. Practically speaking, the 2.5-gee floor applies to all aircraft where *MTOW*>45,000-lbm. Under these rules a 5,000-lbm aircraft would need to be stressed to anywhere between 3.5 and 4.3-gee's at *VL* depending upon power loading. [21]

Condition II represents flight at the maximum level flight speed (*VL*) where the gust load factor is associated with a -30 ft/sec disturbance at the *VL* speed. [22]

Condition III represents flight at the maximum gliding speed (*Vg*) where the design load factor is not less than 2.0-gee's or the greater of 1-gee plus a gust load factor based upon a +15 ft/sec gust at the *VL* speed and 1-gee plus 60% of the manuvering load factor used for condition I. [23]

Condition IV represents flight at maximum glide speed (*Vg*) where the gust load factor is associated with a -15 ft/sec disturbance at the *Vg* speed. [24]

Condition V represents inverted flight (nominally *Nz*= -1.0-gee) flight at maximum level flight speed (*VL*) with a design *nZmax* no less negative than -1.5-gees or -1.0-gee minus the 50% of gust load factor used for condition I and -1.0-gee minus 25% of the maneuvering load factor used for condition I. [25]

1. Condition	I	II	III	IV	V	VI
2. Reference Part 04	.2131	.2132	.2133	.2134	.2135	.2136
3. Design Speed (See § 04.211)	V_L	V_L	V_g	V_g	V_L	V_g
4. Gust Velocity, *U, fps* (1) (2)	+30	−30	+15	−15	--------	--------
5. {Δn (a) Gust (3)	§ 04.2121	§ 04.2121	§ 04.2121	§ 04.2121	$-0.5\Delta n_{Ia}$	--------
5. {Δn (b) Maneuvering	Fig. 04-3	--------	$0.6\Delta n_{Ib}$	--------	$-0.25\Delta n_{Ib}$	--------
6. *Limit* Load Factor, *n*, When line 5 gives two values of Δn, use larger	$1+\Delta n_I$	$1+\Delta n_{II}$	$1+\Delta n_{III}$	$1+\Delta n_{IV}$	$-1+\Delta n_V$	--------
7. Minimum value of *n*	2.50	None	2.00	None	−1.5	None
8. Minimum Yield Factor of Safety, j_y	1.0	1.0	1.0	1.0	1.0	1.0
9. Minimum Ultimate Factor of Safety, j_u	1.5	1.5	1.5	1.5	1.5	1.5

(1) Feet per second.
(2) + means upward, − means downward.
(3) May be limited by maximum dynamic lift coefficient obtainable under sudden changes of angle of attack.

FIGURE 3 - Wing Design Load Conditions – from the 1938 CFR [17]

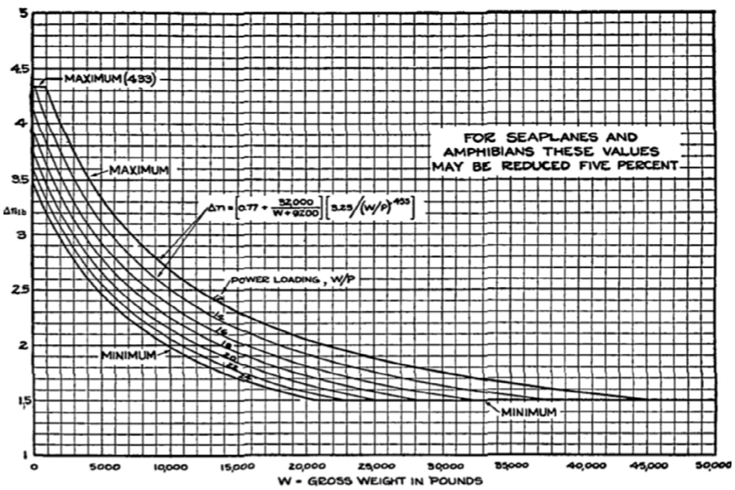


FIGURE 4 - Wing Maneuvering Load Factor Increment [26]

1. Condition	Maneuvering
2. Part Reference 04	.2211
3. Design Speed (See §04.211)	V_p
4. Force Coefficient, C_N	{−.55 (down) / +.35 (up)}
5. Average *Limit* Pressure, p. s. f.(1)	$C_N q_p$ (2)
6. Chord Distribution	Fig. 04-5
7. Span Distribution	Constant C_N
8. Minimum Average *Limit* Pressure, p. s. f.(1)	15
9. Special Requirements	None
10. Minimum Yield Factor of Safety, j_y	1.0
11. Minimum Ultimate Factor of Safety, j_u	1.5

FIGURE 5 - Horizontal Tail Maneuvering Load Factor [18]

1. Condition	Maneuvering
2. Part Reference 04	.2220
3. Design Speed (See § 04.211)	V_p (3)
4. C_N or Gust	C_N=0.45
5. Average *Limit* Pressure, p. s. f.(1)	$C_N q_p$ (2)
6. Chord Distribution	Fig. 04-5
7. Span Distribution	Constant C_N
8. Minimum Average *Limit* Pressure, p. s. f.(1)	12
9. Special Requirements	§ 04.2220 (*b*)
10. Minimum Yield Factor of Safety, j_y	1.0
11. Minimum Ultimate Factor of Safety, j_u	1.5

FIGURE 6 - Vertical Tail Maneuvering Load Factor [19]

14 CFR § 04 required the empennage and control surface to withstand loads from the aforementioned conditions in symmetrical flight (i.e. flight at negligable sideslip) as well as unsymmetric flight (at sideslip). The 1938 CFR provides only vague guidelines "pending the development of more rational methods." [27] For the horizontal tail it stipulates that the design limit loads must incorporate the maximum expected aerodynamic panel load (suggested in the regulations to be -0.55<*CN*<+0.35) encountered during flight at the design manuevering speed (*Vp*) when dimensionalized to be not less than 15-lbf/ft^2. [28] Similarly, the vertical tail must address the maximum expected aerodynamic panel load (suggested in the regulations to be -0.45<*CN*<+0.45) encountered during unsymmetric flight at the design manuevering speed (*Vp*) when dimensionalized to be not less than 12-lbf/ft^2. [29]

The 1938 CFR also stipluates that limiting loads used for the wing structural sizing will comprise air loads and inertia loads "distributed and applied in a manner closely approximating the actual distribution in flight." [30]

The 1938 CFR makes no such callout for inertial relief for empennage and control surface structures; this is consistent with the ideas that empennage and control surface sizing are driven by aerodynamic loads resulting from maximum defleciton at a prescribed manuevering airspeed (*Vp*) which may be considerably lower than maximum level speed (*VL*) or the maximum glide speed (*Vg*).

We see that these structural sizing guidelines from the age of the DC-3 prove remarkably durable; the basic framework governing a defining airspeed-load-factor envelope for wing structural sizing and local-panel-load at airspeed target for empennage and control surface sizing exists to this day.

B. 1940 Amendments

The Civil Aeronautics Board revised structural certification requirements on May 15, 1940. [31] The changes override some of the earlier empirical formulas with an expectation that the airframe designer will be better able to estimate local panel loads. In particular, the board revised 14 CFR § 04.2211 so that the aerodynamic limit loads for the horizontal tail could be limited by those achievable based on 200-lbf pull on the elevator stick. [32]

C. 1945 Amendments

The Civil Aeronautics Board further revised structural certification requirements effective Feb 13, 1945. [33]

These revisions introduce the *V-n* diagram in its present form and fully encapsulate the modern approach to structural sizing.

The 1945 CFR clarifies under 14 CFR §04.200 that "strength requirements are specified in terms of limit and ultimate loads. Limit loads are the maximum loads anticipated in service. … when not otherwise described, loads specified are limit loads. Unless otherwise provided, the specified … loads shall be placed in equilibrium with inertia forces, considering all items of mass in the airplane. All such loads shall be distributed in a manner closely approximating or conservatively representing actual conditions. If deflections under load would significantly change the distribution of external or internal loads, such redistribution shall be taken into account." [34]

The design factor-of-safety and certification test requirement regulations no longer hedge between yield and ultimate. The CFR states unequivocally under 14 CFR § 04.201 that "the factor-of-safety shall be 1.5 unless otherwise specified;" [35] whereas 14 CFR § 04.202 requires structure to be "capable of supporting limit loads without suffering detrimental permanent deformations. At all loads up to limit loads the deformation shall be such as not to interfere with safe operation of the airplane. The structure shall be capable of supporting ultimate loads without failure for at least 3 seconds" [36] by a test which 14 CFR § 04.203 requires to represent "all critical loading conditions." [37]

This amendment defines a clear strategy for the engineer to define certification loads; 14 CFR § 04.210 requires that "flight load requirements shall be complied with at critical altitudes … at all weights between the minimum design weight and design take-off weight with any practicable distribution of disposable load within prescribed operating limitations stated in the airplane operating manual." [38] 14 CFR § 04.2130 stipulates that "a sufficient number of points on the maneuvering and gust envelopes shall be investigated to insure that the maximum load for each member of the airplane structure has been obtained. … all significant forces acting on the airplane shall be placed in equilibrium in a rational or conservative manner … (including) linear inertia forces in equilibrium with wing and horizontal tail surface (aerodynamic) loads." [39]

This concept of a manuevering and gust "envelope" supports the graphical foundation of the *V-n* diagram and associated gust envelope. These novations are introduced in 14 CFR § 04.2111 which holds that "the airplane shall be assumed to be subjected to symmetrical maneuvers resulting in the following limit load factors except where limited by maximum (static) lift coefficients:

(a) The positive maneuvering load factor, *n*, at all speeds up to *Vd*. The value of *n* shall be selected by the designer except that it shall not be less than 2.5.

(b) The negative maneuvering load factor shall have a minimum value of -1.0 at all speeds up to Ve; and factors varying linearly with speed from the specified value at *Vc* to 0.0 at *Vd*." [40]

The 1945 amendements introduce the modern nomenclature of key structural design speeds; these speeds "shall be chosen by the designer except that they shall not be less than the following values:

…

Vf (design flap speed)=1.4 *Vs1* … where *Vsl*=stalling speed, flaps retracted at design landing weight

…

Vp (design maneuvering speed) $=Vs1/\sqrt{n}$ where *n*=limit maneuvering load factor used in design … where *Vs1*=stalling speed with flaps retracted at design take-off weight

…

Vc (design cruising speed)

…

Vd (design dive speed)." [41]

The 1945 regulations provide a floor for both positive and negative design load factors. Regulation 14 CFR § 04.2111 states that the designer shall select a maximum manuevering load factor, *n*, to be used to justify loads "at all speeds up to *Vd*. … it shall not be less than 2.5. … The negative maneuvering load factor shall have a minimum Value of -1.0 at all speeds up to *Vc*; and factors varying linearly with speed from the specified value at *Vc* to 0.0 at *Vd*." [41] (see FIGURE 7)

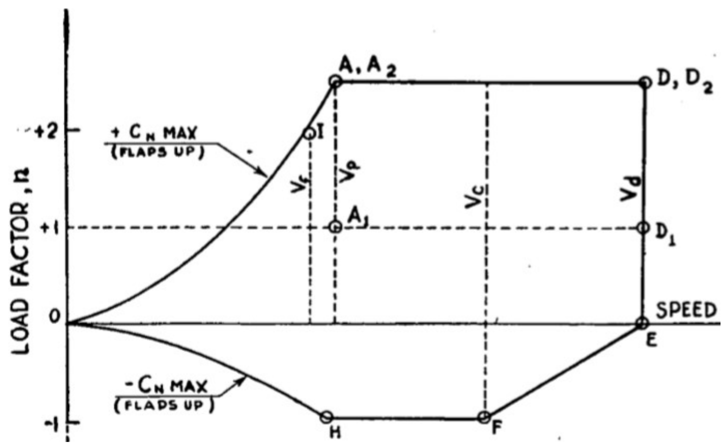


FIGURE 7 - The *V-n* diagram from 14 CFR § 04.2110 (1945) [41]

D. Moden Refinements in 14 CFR § 25

With only minor refinements, the *V-n* diagram has been the mainstay for structural certification of commercial aircraft since 1945.

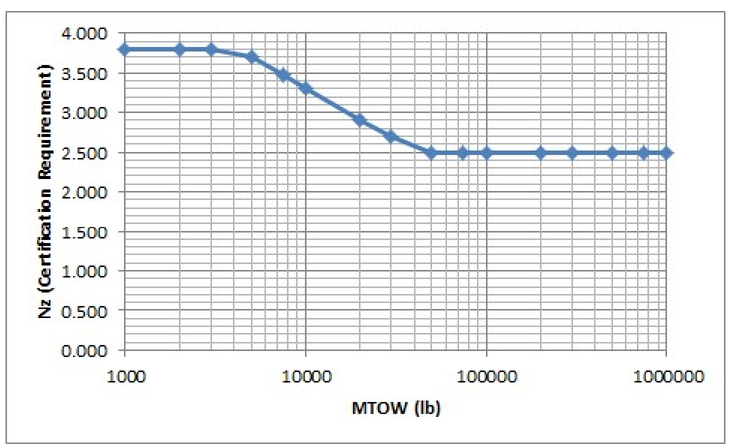


FIGURE 8 – *nZcert* vs Weight limit from 14 CFR § 25.337 [44]

Today, as in 1945, "strength requirements are specified in terms of limit loads (the maximum loads to be expected in service) and ultimate loads (limit loads multiplied by prescribed factors of safety). …. Unless otherwise provided … loads must be placed in equilibrium with inertia forces, considering each item of mass in the airplane. These loads must be distributed to conservatively approximate or closely represent actual conditions. … If deflections under load would significantly change the distribution of external or internal loads, this redistribution must be taken into account." [42]

The design factor-of-safety continues to be "1.5 unless otherwise specified." [43] Strength regulations continue to require structure to be "able to support limit loads without detrimental permanent deformation. The structure must be able to support ultimate loads *(i.e. limit loads multiplied by the 1.5 factor-of-safety)* without failure for at least 3 seconds."

Modern transport category regulations provide a revised floor for both positive and negative design load factors.Modern regulations continue to limit the peak positive and negative load factors. Modern 14 CFR § 25.337 stipulates that "the positive limit maneuvering load factor n for any speed up to Vn may not be less than 2.1 + 24,000/(W + 10,000) except that n may not be less than 2.5 and need not be greater than 3.8 - where W is the design maximum takeoff weight … (and) the negative limit maneuvering load factor - May not be less than −1.0 at speeds up to *Vc*; and must *VA*ry linearly with speed from the value at *Vc* to zero at *Vd;*" (see FIGURE 8). [44]

E. A regression to 1920's style language in the revised 14 CFR § 23

In 2016, the FAA (with consent of its European counterpart EASA) promulgated a complete re-write of airworthiness standards for newly certified general aviation aircraft. The "new" Part 23 revises "the existing prescriptive certification standards to performance-based standards for a number of aspects of an airplane's design. … (intended) to maintain the level of safety currently provided through the existing Part 23 requirements." [45] In reality, the regulatory changes result in a legal roll-back to 1926-style vague language that the 1938 and 1945 as well as later amendments would clarify.

The revised 14 CFR § 23 structural standards now define limit loads as those "which are equal to the structural design loads … (and) ultimate loads, which are equal to the limit loads multiplied by a 1.5 factor-of-safety." [46]

The revised 14 CFR § 23 structural standards define the structural design envelope as" the range and limits of airplane design and operational parameters for which the (designer) will show compliance." [47] This includes "structural design airspeeds … any other airspeed limitation(s)." [47] as well as "design maneuvering load factors not less than those, which service history shows, may occur within the structural design envelope." [47] Flight load conditions must consider "atmospheric gusts … symmetric and asymmetric maneuvers." [48]

F. Sum Up

From the very inception of regulated aircraft construction, design flight loads must consider the entire operational envelope of the aircraft – both in terms of speed and altitude as well as weight with a variety of loading conditions from flight at operational empty weight to flight at maximum takeoff weight. We see how some loads, particularly those governing the structural sizing of the main wing spar, are driven by the maximum design load factor (*nZcert*); *nZcert* is typically defined in terms of gees at the maximum certification weight possibly tempered by gust response. We also see how control surface and empennage loads are driven by a maximum dynamic pressure condition, typically the *VA* maneuvering speed for control surfaces and *Vd* for the empennage.

It is interesting that the codification of the 1.5 factor-of-safety dates from a time when the understanding of flight loads was crude, at best. In 1945, engineers did not well understand stress-corrosion-cracking and other aging related topics which impact fatigue, damage tolerance and service life. While current aircraft exhibit cracking and other loading derived damage during their operational life, perhaps it would be best to revisit the reason why a "1.5 factor-of-safety" works as well (or as poorly) as it does. In the future, it might make more sense seeing that we can characterize loads and materials to call out a distinct factor-of-safety regarding the uncertainty of loads separate from a derate on materials properties due to fatigue or materials aging concerns.

III. Where Aerodynamic and Structural Certification Basis Intersect

A. Aerodynamic Regulatory Basis

Civilian aerodynamic certification limits must consider flight at a variety of weights, from the maximum weight selected by the applicant through a minimum weight, "not less than … the design minimum weight." [49]

The aircraft must also be "safely controllable and maneuverable during - (1) Takeoff; (2) Climb; (3) Level flight; (4) Descent; and (5) Landing. … without exceptional piloting skill, alertness, or strength, and without danger of exceeding the airplane limit-load factor … including (as a result of) the sudden failure of the critical engine." [50]

Reserve control power must be sufficient to permit "20° banked turns, with and against the inoperative engine, from steady flight at a speed equal to 1.3 *VS*.." [51] If the aircraft exceeds its speed, beyond "*VMO*/*MMO* … it must be shown that the airplane can be recovered to a normal attitude and its speed reduced to *VMO*/*MMO*, without … exceptional piloting strength or skill; or exceeding *VD/MD,VDF/MDF*, or the structural limitations and buffeting that would impair the pilot's ability to … control the airplane for recovery." [52]

Minimum scheduled flight speeds should be no slower than 1.13 times the stall speed or the minimum control airspeed, *VMC*, which is the "airspeed at which, when the critical engine is suddenly made inoperative, it is possible to maintain control of the airplane with that engine still inoperative and maintain straight flight with an angle of bank of not more than 5 degrees."[53]

Thus, certification bounds define limiting structural design airspeed (*VD*) and Mach numbers (*MD*) – consistent with the *V-n* diagram which are slightly above the maximum design airspeeds (*VMO/MMO*). This provides an upper bound to the dynamic pressure and Mach number found in scheduled flight. Functionally, *VD* provides a "structural flight demonstration speed" in an intentional dive where the airframe must maintain integrity from static loads and remain free of flutter. In normal operation, pilots should not fly above the scheduled VC speeds.

B. Fundamental Aerodynamic Limits of Transonic Aircraft Do Not Support a Literal *V-n* Diagram

Certification also bounds the low speed limits, of the 1-gee stall speed (the slowest stall speed will correspond to flight at the design minimum weight) not less than the lateral-trim-limits imposed by *VMCA*.

While the "*V-n*" diagram is considered a certification basis point at the design maximum weight, the realities of the maneuvering speed are functions of the flight weight.

Consider first, that Mach effects mean that the usable *CLmax* (limited by both trim, stability and buffet limits is typically a strong function of Mach number; see FIGURE 9. [54] Note that the *VA* speed in KEAS is a function of the design *nZcert* and the stall speed (which in turn is a function of flight weight):

$$Vs = 660.8\sqrt{\frac{\left(\frac{1}{CLmax}\right)\left(\frac{MTOW}{Sref}\right)}{1481}} \qquad (2)$$

Therefore, the *VA* speed based on a low-speed *CLmax* is:

$$VA = Vs \cdot \sqrt{nZ_{cert}} \qquad (3)$$

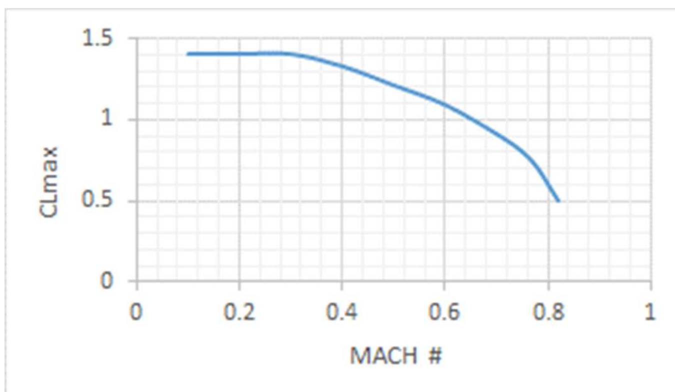


FIGURE 9 - *CLmax* as limited by *CLbuffet* for a notional narrow-body airliner

We can highlight the disconnect implicit in the simplistic *V-n* diagram with the following example.

Consider a narrow body airliner with low speed *CLmax*=1.4, *Sref*=1319-ft^2 certified at a flight weight *MTOW*=175,000-lbm, following equation (2), we see that it has a notional low-Mach stall speed of 167-KEAS. If it is certified to *nZcert* = +2.5-gees, then following equation (3), the *VA* speed will be 249-KEAS.

In reality, the buffet boundary will limit *CLmax* with Mach number as seen in FIGURE 9.

Next consider flight at a typical cruise condition: *M*=0.80 at FL350. Now, buffet conditions limit *CLmax* to ~0.6; so the practical stall speed is now 255-KEAS. Flight at *W*=175,000-lbm under these conditions is flight at 257-KEAS and corresponds *CL*~0.59; therefore, the maximum attainable load factor limited by aerodynamics is (0.6/0.59)=1.01-gees. In reality, heavily loaded, this aircraft is unlikely to cruise at FL350 due to incipient buffet.

FIGURE 10 explains the tendency for cruising transport aircraft to descend and slow slightly when encountering light chop at altitude; typical peak efficiency occurs close to the buffet limit – whereas flight at a somewhat slower speed and lower altitude provides significantly more buffet margin (aerodynamic *nZmax*) against upsets.

Fundamentally, there is nothing inherent in the mathematics that requires the *Vc* speed to be above the low speed *VA* speed. For example, if we were to re-contour the wing of our hypothetical transport to have a low speed *CLmax*~1.6 without loss of high-speed performance, its notional Low-Mach stall speed at the maximum certification weight would decrease to 156-KEAS and its *VA* speed would decline to 247-KEAS – slower than its typical *M*=0.8/FL350 *Vc*/*MC* cruise point.

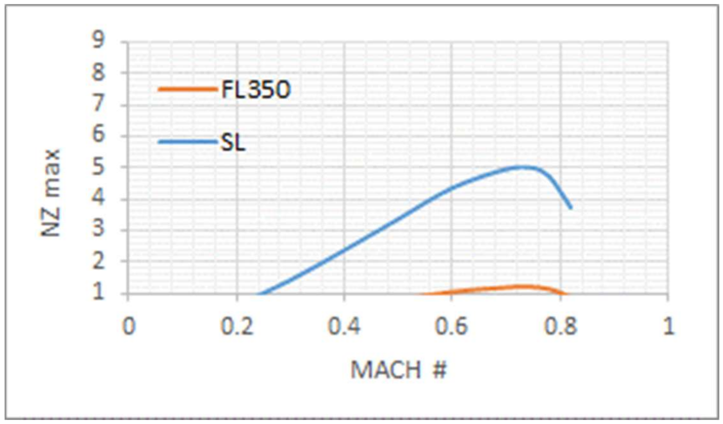


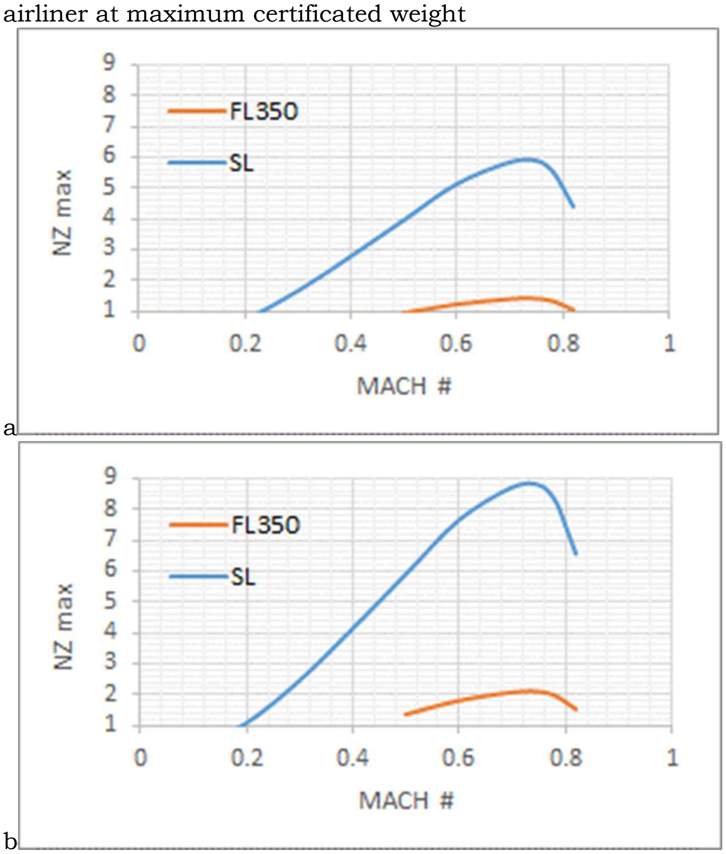
FIGURE 10 - Aerodynamic *nZ* max limits of a narrow-body airliner at maximum certificated weight

It is important that this trend means the wing is designed conservatively (over-designed for High-Mach maneuverability. i.e. at least the structure is trending the right direction.

However, this is another instance where sizing for dynamic load limits immediately at takeoff is overly-conservative for an airplane maneuvering up high after it has burned off considerable fuel.

C. Flight Weight Impacts of Aerodynamically Attainable Load Factor

a



b

FIGURE 11 - Aerodynamic *nZ* max limits of a narrow-body airliner at: a) 150,000-lbm b) 100,000-lbm

We can repeat this process at sea-level; and for differing flight weights. Refer back to FIGURE 10, where we saw the strong altitude dependence of attainable load factor. Turn next to FIGURE 11, where we see that at *W*=150,000-lbm and *M*=0.8 the aircraft has an aerodynamically attainable *nZmax* = 1.18 at FL350 (flight at 257-KEAS) and *nZmax* = 5.0 at sea-level (flight at 529-KEAS). At *W*=100,000-lbm and *M*=0.8 the aircraft has an aerodynamically attainable *nZmax*=+1.77 at FL350 (flight at 257-KEAS) and *nZmax*=+7.5 at sea-level (flight at 529-KEAS). Thus across a variety of weights, we see that this aircraft at FL350 has a *Vc* speed very close to its *VA* speed but flight at lower altitudes will considerably exceed its *VA* speed. Equally important is the trend that the aerodynamically attainable load factor increases in opposition to the flight weight; half the weight means double the gee's.

We must next what these aerodynamically attainable *nZmax* limits mean in terms of structural limits. Recall, that there are two driving factors regarding the structural limit: 1) is wing transverse bending (defined in terms of gees at the maximum certification weight) and 2) how control surface and empennage loads driven by a maximum dynamic pressure conditions. Thus we must consider two separate static loading conditions which may temper the flight envelope. Note that the final design must also demonstrate integrity under dynamic loading conditions (i.e. freedom from flutter).

Let us begin by considering the wing transverse bending case. In practice, the defining *nZmax* from the *V-n* diagram functionally specifies a particular bending moment distribution. The structure must withstand tensile loads, compressive loads and shear loads associated with these torques. In the absence of any inertial relief, the bending moment will directly scale with the inertial load, represented by the load factor, *nZ*, and the flight weight.

$$MOMENT \propto nZ \cdot W \tag{4}$$

Such a wing will exhibit the following trend:

$$Nzmax = \frac{nZ_{cert} \cdot W}{MTOW} \tag{5}$$

In reality, the bending moments will be offset by some inertial load relief which is greater at high flight-weights (i.e. when the wing is loaded with fuel) than at light flight-weights. Thus the design bending moments will be offset by some factor

$$MOMENT \propto nZ \cdot W \cdot (1 - BMR) \tag{6}$$

Where the bending moment relief factor, BMR, will be at its greatest at *MTOW* and reach zero at a typical zero-fuel weight.

Thus in terms of design bending moments:

$$nZ_{limit} \cdot W = nZ_{cert} \cdot MTOW \cdot (1 - BMR) \tag{7}$$

We may thus, develop an empirical chart based upon various BMR trends. Consider our narrow-body transport, assume that the aircraft carries 40-000-lbm of fuel under typical *MTOW* conditions (that is the fuel load is ~23% of the maximum takeoff weight). If this fuel load reflects a peak of 16% reduction in wing bending moment at *MTOW* and declines to 0% impact at zero fuel load; we can estimate the *nZmax* vs *W* trends for equivalent root-bending moment as we "burn-off" fuel from *MTOW* to our heavy *ZFW*; see FIGURE 12. There is a small, but appreciable change in structural load capacity – in the absence of any design credit for bending moment relief our +2.5-gee certified aircraft is strong enough to withstand +3.13-gees at 140,000-lbm – with bending moment relief considered in the initial structure, the aircraft can express 2.68-gees at 140,000-lbm. Next, consider this airplane operating with a moderate fixed payload, so that its zero-fuel-weight is ~110,000-lbm. We see that with "full tanks" – at a flight weight of ~155,000-lbm – a load factor of +2.82-gees (at 16% BMR) matches the +2.5-gee (at 16% BMR) @ *MTOW* design bending torques. As the aircraft further burns off fuel, the aircraft can attain up to +3.34-gee's of load factor at zero fuel without violating the design bending moment constraint. Finally, consider this airplane operating with a minimal fixed payload, so that its zero-fuel-weight is near its *OEW* of ~90,000-lbm. We see that with "full tanks" – at a flight weight of ~140,000-lbm – a load factor of 3.13-gees (at 16% BMR) matches the 2.5-gee (at 16% BMR) @ *MTOW* design bending torques. As

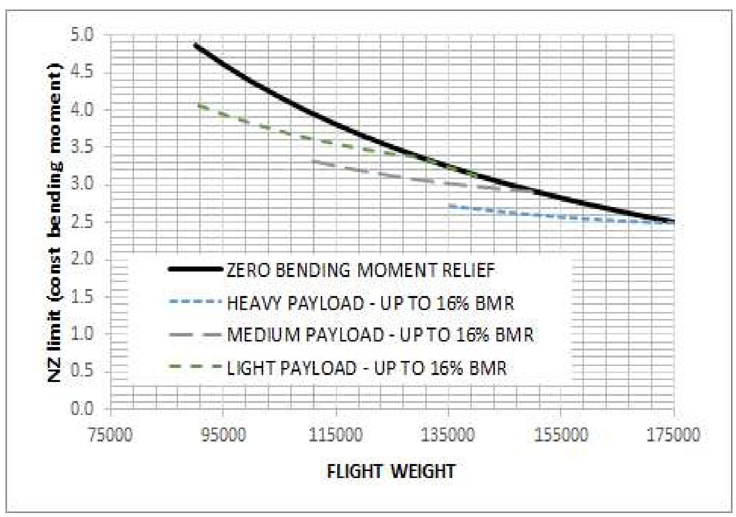


FIGURE 12 - *NZlimit* as a function of payload and flight weight; notional *NZcert*=+2.5-gee @ 175,000-lbm.

the aircraft further burns off fuel, the aircraft can attain up to 4.08-gee's of load factor at *OEW* without violating the design bending moment constraint. Thus, we can see how "2.5-gee" certified airframes can easily fly nearly 4-gee maneuvers under light-weight, "air-show" conditions without reinforcement or structural damage.

We may now return to our aerodynamic *nZmax* charts and consider the combined effects and implications of aerodynamics (including buffet) with 1) max KEAS limitations, 2) maximum Mach number limitations and 3) a flight weight dependent *Nz* limit. For purposes of example, let us continue with the notional narrow-body transport aircraft with *Vd*=350-KEAS and MD=0.82. At *W=MTOW*=175,000-lbm, *nZlimit* = *nZcert* = +2.5-gees; at *W*=150,000-lbm (86% MTOW), *nZlimit* = +2.92-gees.; *W*=125,000-lbm (71% MTOW), *nZlimit* = +3.42-gees and at *W*=100,000-lbm (57% MTOW), *nZlimit*=+3.83-gees. FIGURE 13 shows the progression of maneuvering capability; at all weights the *VA* speed can be attained at sea-level but the *Nz* associated with the *VA* speed increases as the flight weight decreases. At FL200, the interaction between Mach buffet an *Nz* limits presents itself; at the lightest weights this aircraft has the aerodynamics to attain *nZlimit* at FL200 prior to buffet onset whereas at heavier weights it cannot. At FL300 and FL350 the aircraft does not have enough aerodynamic capability to attain *nZmax*. These trends are the manifestation of situations where the *Vd* speed (as defined by high-speed aerodynamics) falls below the *VA* speed (as defined by low speed aerodynamics) effectively "truncating" the usable limits of the *V-n* diagram.

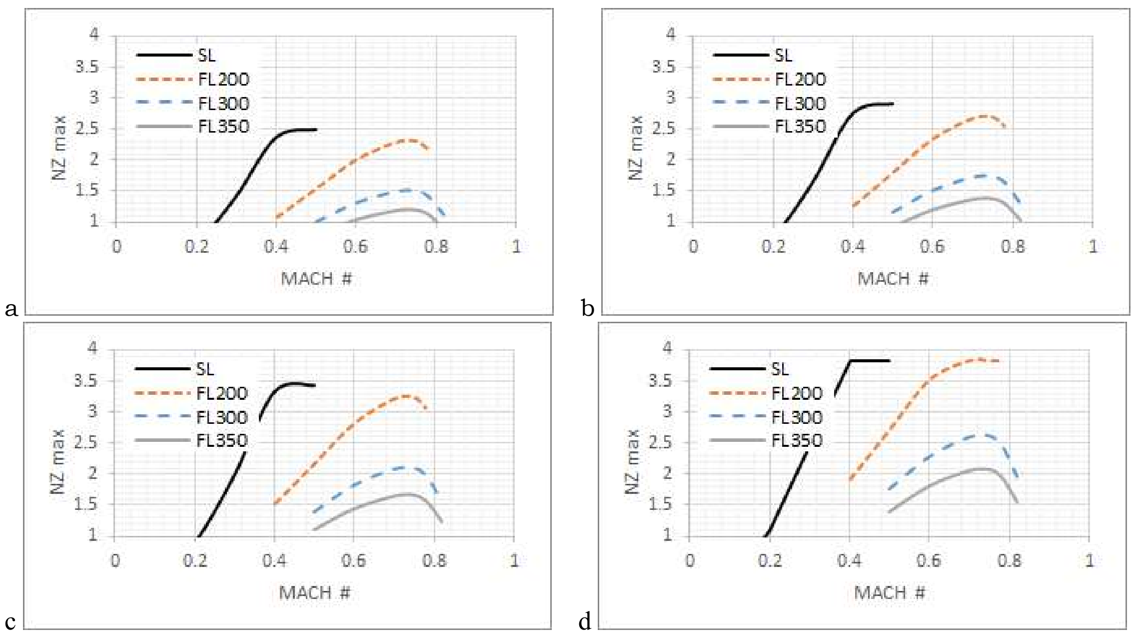


FIGURE 13- Maneuvering Limits of a Narrow-Body Airliner including Max Mach (*MD*) and maximum KEAS (*VD*) and flight weight dependent *nZlimit*. a) 175000-lbm, b) 150000-lbm, c) 125000-lbm, d) 100000-lbm

The Navy test pilot school Fixed Wing Performance Manual [55] notes that this is a common characteristic of high-speed, high performance aircraft; see FIGURE 14. They note that while the *V-n* diagram is typically drawn with "sharp" corners, it may prove difficult or impossible to document with flight test the precise boundary imposed by buffet limits in load factor-vs-Mach number space.

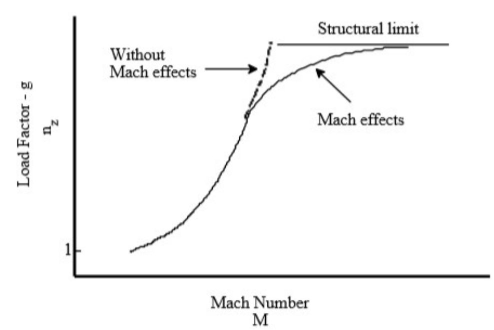


FIGURE 14 - Compressibility effects on maximum attainable load factor, *nZlimit* after Ref [55]

IV. Turn Capability and Agility Metrics

This section defines and derives common and novel metrics regarding airplane turning performance and agility characteristics. Recall that an airplane in flight has a velocity vector which defines both its speed and direction of flight. Maneuverability is the capability for an aircraft to change this vector. In order to quantify aircraft maneuverability, we must determine its linear acceleration, climb deceleration, and turning characteristics. Also remember that aircraft lack inherent pendulum stability. This is because the aerodynamic center of lift is nearly always longitudinally and laterally coincident with the aircraft's center of gravity. Any vertical displacement is at best small. In fact, since most aircraft have their wings mounted below their center of gravity, they actually have weak pendulum instability.

When maneuvering, pilots alter the forces of lift, weight, thrust, and drag a to generate linear or radial accelerations. As aircraft lack an inherent tendency to flight right-side up, they typically fly bank to turn flight profiles. Pilots command heading changes by rolling the aircraft to the left or right in order to tilt the direction of the lift vector. To finish the turn, the pilot rolls back to the wings-level position. The radial acceleration developed by the tilted lift vector causes a turn in the horizontal, vertical, or even in an oblique plane. Forces which cause a radial acceleration include: weight, side force, lift, and thrust. If the aircraft is to turn without a loss of altitude, the vertical component of the lift force must continue to equal the weight. Thus, the pilot must pull back on the stick to increase lift to an amount greater than the weight of the aircraft. The horizontal component is unbalanced; this force (balanced by centrifugal force) causes the aircraft to accelerate inward and execute the turn.

A. Instant Turn Radius and Rate

The Navy Test Pilot School Fixed Wing Performance Manual [55] suggests that the primary characteristics which describe an aircraft's instantaneous turn capability are its turn radius and heading-change turn rate. The instantaneous turn performance describes the capability of an airplane at a particular flight condition, at an instant in time. We do not consider the airplane's ability to sustain the performance for any length of time. In fact, the energy loss rate may be high – a turn at maximum instantaneous turn rate is often accompanied by rapid deceleration or altitude loss. But first, let us consider the maneuvering rotation of the airframe alone – as governed by its aerodynamics and structures.

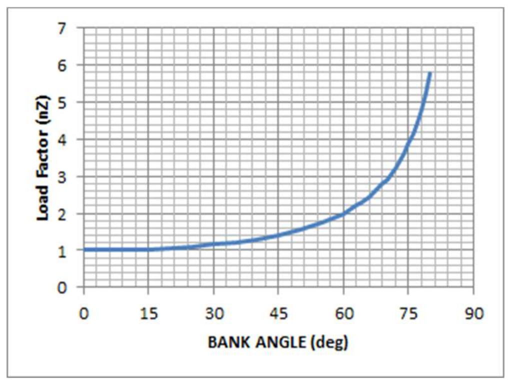


FIGURE 15 –Bank Angle / Load Factor Relationship

As discussed earlier, the attainable load factor, *nZ*, represents the magnitude by which lift exceeds weight limited by structural concerns:

$$nZ = \min\left(\frac{W}{CLmax\ q\ Sref}, nZlimit\right) \quad (8)$$

Where both the dynamic pressure, q, and the maximum lift coefficient, *CLmax*, are functions of speed and altitude.

Geometry also implies a correlation between *Nz* and bank angle, Φ, for flight without loss of altitude (FIGURE 15):

$$nZ = 1/\cos(\Phi) \quad (9)$$

Turning radius in nautical miles may be inferred from load factor, *Nz*, and flight speed in KTAS, where g = 32.2-ft/sec^2 (FIGURE 16)

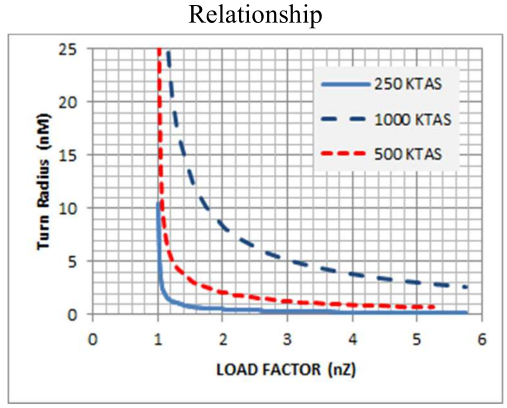


FIGURE 16 –Turn Radius as a function of Load Factor and Flight Speed

$$TURNRADIUS = \frac{(V)^2}{g\sqrt{nZ^2-1}} = \frac{(VKTA \cdot \frac{6076}{3600})^2}{32.2\cdot\sqrt{nZ^2-1}} \Big/ 6076 \qquad (10)$$

At the 1-gee stall speed, no turns can be made; wings must be held level – the turn radius is infinite and turn rate is zero.

As we increase airspeed above the stall speed, the allowable bank angle increases and the turn radius rapidly diminishes. The turn radius increases as a function of the true airspeed squared; the reader may examine the trade between load-factor and speed in FIGURE 16. For example, a supersonic fighter aircraft with a 5 gee load factor at 1,000 KTAS true airspeed, will bank over 78° to make a ~3-nM radius turn. A subsonic transport can fly the same course and make the same 3-nM radius turn at 250 KTAS with only a 17° bank angle and a barely perceptible load factor of 1.045! If we utilize 2.5-gees of load factor, the transport category aircraft will bank to 66° and develop an instantaneous turn radius of 0.4-nM at 250 KTAS.

We may also consider the instantaneous rate-of-heading change capability. Recall that the arc length of a circle is 2π times its radius. Thus, if we know the turn radius in feet and we know the flight speed in true-airspeed we can infer the turn rate in terms of degrees-of-heading-change per second:

$$TURNRATE = 360 \Big/ \left(2\,\pi \frac{TURNRADIUS}{VKTAS\frac{6076}{3600}}\right) \qquad (11)$$

That transport aircraft with a turn radius of 0.4-nM (~2,500-ft) at 250 KTAS develops a heading rate change of ~9.7°/sec; the supersonic fighter at 5-gees and 1000 KTAS will only turn at ~5.3°/sec. Thus the transport actually demonstrates considerably greater agility and maneuverability because of its low flight speed and in spite of its limited gee capability.

B. Sustained Turn Radius and Rate

Performance engineers like to qualify the sustained turn capability inherent in an airframe. That is its ability to maintain a steady turn without loss of speed or altitude.

In order to compute sustained turn capability (both in terms of load factor, the implied bank angle, turn radius and turn rate) we must pay attention to the specific excess power of the airframe. Thus, sustained turn capability, *nZsustained*, reflects equation (8) where *CLmax* may also be limited by the lift coefficient associated with thrust equaling drag (*T=CD q Sref*) at a given power setting. The value of *nZsustained* is found using a computer program employing root-finding algorithm which attempts to balance thrust and drag for flight at a given speed/altitude pairing within the limits of the *CLmax*.

Once *nZsustained* has been established, we can use equations (9), (10) and (11) to compute the implied bank angle, turn radius and turn rate.

The Navy Test Pilot School Fixed Wing Performance Manual [55] presents some illustrative charts to describe typical combat aircraft sustained turn capability. These plots, seen in FIGURE 17, must be developed at a representative flight weight and altitude. We can see that for any given weight, speed and altitude combination, the sustained turn capability can never exceed the instantaneous turn capability of an airframe. The sustained turn capability tracks the instantaneous turn capability from stall speed up to some limiting speed,

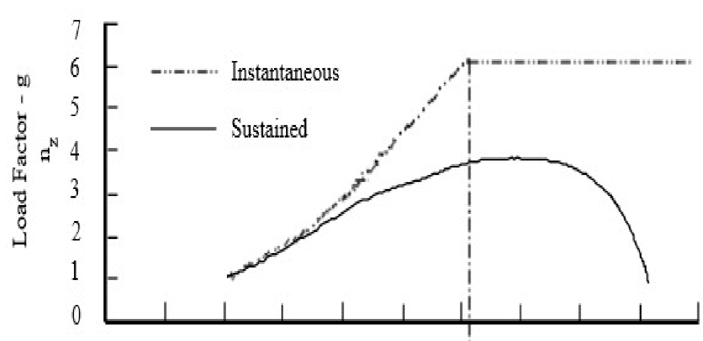


FIGURE 17 –Load factor as a function of Mach number schematic, after Ref [55]

typically far below the *VA* speed. As the Mach number further increases, sustained turn capability diminishes.

C. Unsteady Turning Flight

The Navy Test Pilot School Fixed Wing Performance Manual [55] suggests a number of metrics to document unsteady flight.

- The first area of interest is the 1-gee speed envelope for level flight; we seek to document the acceleration potential of the airplane in knots per second at full "military" thrust.
- The second area of interest comprises performance in windup turns. A windup turn involves a smooth and steady increase in load factor flown at constant Mach number beginning at 1-gee level flight and ending at the stall or buffet limit; the aircraft may climb or descent at various points in this maneuver. We will document this in terms of unaccelerated-rate-of-climb in ft/min across the flight envelope.
- The third area of interest involves turns at constant altitude. The "front side" technique documents the acceleration potential of the airplane in knots per degree heading change and full "military" thrust.
- The final area of interest involves turns at constant speed. The "back side" technique documents the unaccelerated-rate-of-climb in ft per degree heading change at full "military" thrust.

We calculate the acceleration potential is 1-gee level flight as:

$$ACCEL = 19.078\,\frac{(T - CD\;q\;Sref\,)}{W} \tag{12}$$

where *ACCEL* is in KTAS/sec, thrust, *T*, is given in lbf, dynamic pressure, *q*, in lbf/ft^2 and wing reference area, *Sref*, in ft^2. *CD* reflects flight where $CL = W\,/\,(\,q\;Sref\,)$.

The unaccelerated-rate-of-climb is:

$$ROC = 101.33\,\frac{(T - CD\;q\;Sref\,)}{W}\;KTAS \tag{13}$$

where *ROC* is in ft/min; the thrust, *T*, is given in lbf; the dynamic pressure, *q*, in lbf/ft^2 and the wing reference area, *Sref*, in ft^2. *CD* reflects flight at *CLmax* as limited by stall, buffet or structural limits.

The "front side" acceleration potential is:

$$ACCEL_{FS} = {19.078\,\frac{(T - CD\;q\;Sref\,)}{W}}\Big/{TURNRATE} \tag{14}$$

where $ACCEL_{FS}$ is in KTAS/deg-heading-change, thrust, *T*, is given in lbf, dynamic pressure, *q*, in lbf/ft^2 and wing reference area, *Sref*, in ft^2. *CD* reflects flight at *CLmax* as limited by structure, stall and/or buffet limits. TURNRATE is calculated using Eqn [11] in terms of degrees-heading-change per second.

The "back side" rate of climb is:

$$ROC_{BS} = {1.688\,\frac{(T - CD\;q\;Sref\,)}{W}\;KTAS}\Big/{TURNRATE} \tag{15}$$

where ROC_{BS} is in ft/deg-heading-change, thrust, T, is given in lbf, dynamic pressure, q, in lbf/ft2 and wing reference area, Sref, in ft^2. *CD* reflects flight at *CLmax* as limited by stall, buffet and/or structural limits. TURNRATE is calculated using Eqn [11] in terms of degrees-heading-change per second.

V. Example – Narrow Body Transport Aircraft

For illustrative purposes, let us compute maneuverability characteristics for a narrow-body transport category aircraft. My students and I have developed a calibrated reverse-engineered model [54] of the Airbus A320 CEO using EDET [56] for aerodynamic data prediction and NPSS [57] for propulsive system performance. Prior work by Beard [58] and Wilson [59] establish how closely this model reflects Airbus A320 flight manual "scheduled performance" values. [60]

Note that the A320 is certified for a maximum operating altitude of FL390. It has an *OEW* of ~95,000-lbm and is certified for *MTOW* at ~175,000-lbm. We believe that despite the significant flight-envelope protection features embodied in its active flight control system that the structure was certified to *nZcert* = +2.5-gees at ~175,000-lbm. The *VD* speed appears to be 350-KEAS. [60]

While we postulate the possible aerobatic performance of an A320CEO here, we must note the real A320 has extensive envelope protection including over-speed, over-gee and over-bank-angle limits. The limiting performance described here in many cases would likely trigger envelope protection, so we must consider this sort of performance as being attainable only if the aircraft was flown in "manual" mode.

Note here that all computations are produced assuming flight on a ISA standard day.

We will show combat maneuverability metrics for this airframe both at the notional certification weight (*MTOW*=175,000-lbm) and at a notional "air-show" flight weight (i.e. minimal fuel and no interior) (W=100,000-lbm).

We will compare our performance estimates against a famous "Third-Generation Fighter," the McDonnell F-4E Phantom II.

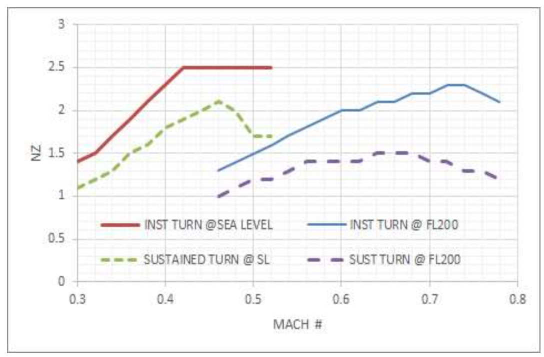


FIGURE 18 - MOCK A320 – Compressibility and Altitude Effects on *nZmax* @ *MTOW*=175,000-lbm

First, let us recreate FIGURE 14 and FIGURE 17 style plots. In FIGURE 18, we see that the A320 produces a classic sharp edged *V-n* diagram near sea-level, but lacks sufficient excess thrust to attain sustained turn capability that matches instantaneous turn capability at any speed. At FL200 compressibility effects soften the maximum instantaneous *Nz* envelope. For A320, its *Vc* speed falls below its low-altitude *VA* speed; the aircraft lacks enough wing and *CLmax* to attain it maximum structural load limits at this altitude. At higher altitudes the disconnect between the notional *VA* speed at cruise conditions grows larger.

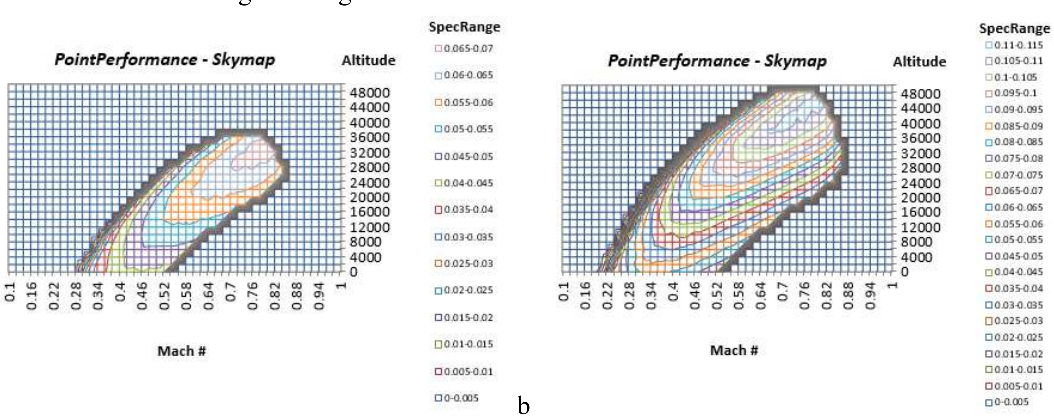


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FIGURE 19 - MOCK A320 – Spec Range (nM/lbm-fuel) as a function of speed and altitude a) 175000-lbm, b) 100000-lbm

Although not expressly called out in the CFR, this sort of behavior is inherent in typical aircraft.

We begin with a "skymap plot" of our reverse engineered A320; these are contour plots of various aircraft performance parameters – a format made popular by Boyd & Sprey [61] which has been embraced by the present author. [54]

The quality of the calibration of the A320 aerodynamic and performance models is revealed in FIGURE 19 "skymap" plots (prior page) of 1-gee level flight specific range; that is the distance covered in Nm per lbm of fuel consumed. We see that the best specific range at heavy weight cruise is ~0.07-nM/lbm which occurs around M~0.78/FL340. This closely approximates the performance found in the A320 flight manual. [60].

At the "airshow" flight weight, W=100,000-lbm, best specific range increases to ~0.115-nM/lbm at M~0.78/FL440. We note that in revenue service, the A320 is unlikely to fly revenue missions below 140,000-lbm. This is reflected in its certification ceiling not to exceed FL390.

Now that we have established that our aerodynamic and propulsive models closely approximate an actual airframe, let us move on to examining its "combat maneuverability" capability.

FIGURE 20 shows the level-acceleration capability of this aircraft when flown in 1-gee flight. We compute acceleration capability using Eq (12) in terms of knots-true-airspeed gained (or lost) per second; KTAS/sec. Peak acceleration capability is found near sea-level. Comparing FIGURE 20a at 175,000-lbm to FIGURE 20b at 100,000-lbm, it is clear that acceleration improves at flight weight decreases. As we rise in altitude, level acceleration declines. As we slow down and approach stall, increasing induced drag also mutes acceleration capability. Finally, if we speed up and enter drag rise, we also lose acceleration capability.

1-GEE LEVEL FLIGHT ACCELERATION CAPABILITY (KTAS/sec)

PRESSURE ALTITUDE (FT) \ MACH (#)	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80
40000																										
38000																										
36000																				0.0	0.1	0.1	0.2	0.2	0.2	
34000																		0.1	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.2
32000																0.1	0.2	0.3	0.3	0.3	0.4	0.4	0.3	0.3	0.3	0.3
30000															0.3	0.3	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.3	0.3
28000													0.1	0.4	0.4	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.4	0.3	0.3
26000												0.2	0.5	0.5	0.6	0.6	0.6	0.6	0.6	0.5	0.5	0.5	0.4	0.3	0.3	0.2
24000											0.4	0.5	0.6	0.6	0.7	0.7	0.6	0.6	0.6	0.5	0.5	0.4	0.4	0.3	0.3	0.2
22000										0.5	0.6	0.7	0.7	0.7	0.7	0.7	0.7	0.6	0.6	0.5	0.5	0.4	0.3	0.3	0.2	0.1
20000									0.6	0.7	0.8	0.8	0.8	0.8	0.8	0.7	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.2	0.1	
18000								0.6	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.7	0.6	0.6	0.5	0.4	0.4	0.3	0.2			
16000							0.6	0.8	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.7	0.6	0.5	0.5	0.4	0.3					
14000						0.5	0.9	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.7	0.6	0.6	0.5	0.4	0.3						
12000					0.3	0.9	0.9	1.0	1.0	1.0	0.9	0.9	0.8	0.7	0.7	0.6	0.5	0.4	0.4							
10000				0.0	0.9	1.0	1.0	1.0	1.0	1.0	0.9	0.8	0.8	0.7	0.6	0.6	0.5									
8000			0.1	1.4	1.7	1.7	1.6	1.4	1.3	1.1	0.9	0.8	0.7	0.7	0.6	0.5										
6000			1.8	2.3	2.4	2.4	2.1	1.8	1.5	1.2	0.9	0.8	0.7	0.6	0.5											
4000		1.7	2.7	2.8	2.8	2.8	2.5	2.1	1.7	1.2	0.8	0.7	0.6	0.5												
2000	1.0	2.7	3.0	3.0	3.1	3.0	2.6	2.2	1.7	1.2	0.8	0.7	0.6													
0	2.4	3.2	3.3	3.3	3.3	3.2	2.7	2.2	1.7	1.2	0.7	0.6														

a

1-GEE LEVEL FLIGHT ACCELERATION CAPABILITY (KTAS/sec)

PRESSURE ALTITUDE (FT) \ MACH (#)	0.30	0.32	0.34	0.36	0.38	0.40	0.42	0.44	0.46	0.48	0.50	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.70	0.72	0.74	0.76	0.78	0.80
40000													0.4	0.7	0.8	0.8	0.9	0.9	0.9	0.9	0.9	0.8	0.8	0.8	0.7	0.7
38000											0.0	0.8	1.0	1.0	1.1	1.1	1.1	1.1	1.1	1.1	1.0	1.0	0.9	0.9	0.9	0.8
36000										0.4	1.1	1.2	1.3	1.3	1.3	1.3	1.3	1.3	1.3	1.2	1.2	1.1	1.1	1.0	1.0	0.9
34000									0.7	1.3	1.4	1.5	1.5	1.5	1.5	1.5	1.5	1.4	1.4	1.3	1.3	1.2	1.2	1.1	1.1	1.0
32000								0.9	1.5	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.5	1.5	1.5	1.4	1.3	1.3	1.2	1.1	1.1	1.0
30000							1.1	1.7	1.8	1.8	1.8	1.8	1.8	1.8	1.7	1.7	1.6	1.6	1.5	1.5	1.4	1.3	1.2	1.1	1.0	0.9
28000						1.2	1.9	1.9	2.0	2.0	2.0	1.9	1.9	1.8	1.8	1.8	1.7	1.6	1.5	1.5	1.4	1.3	1.2	1.1	0.9	0.8
26000					1.2	2.0	2.1	2.1	2.1	2.1	2.1	2.1	2.0	1.9	1.9	1.8	1.7	1.6	1.6	1.5	1.4	1.2	1.1	1.0	0.8	0.7
24000				1.1	2.1	2.3	2.3	2.3	2.3	2.3	2.2	2.1	2.0	2.0	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.1	1.0	0.9	0.7	0.5
22000			0.8	2.2	2.4	2.4	2.5	2.4	2.4	2.3	2.3	2.2	2.1	2.0	1.9	1.8	1.7	1.6	1.4	1.3	1.2	1.0	0.9	0.7	0.6	0.4
20000		0.3	2.1	2.6	2.6	2.6	2.6	2.6	2.5	2.4	2.3	2.2	2.1	2.0	1.9	1.8	1.6	1.5	1.3	1.2	1.1	0.9	0.7	0.6	0.4	
18000		1.8	2.7	2.7	2.7	2.7	2.7	2.6	2.5	2.4	2.3	2.2	2.0	1.9	1.8	1.7	1.5	1.4	1.2	1.1	0.9	0.7	0.6			
16000	1.3	2.7	2.9	2.9	2.8	2.8	2.7	2.6	2.5	2.4	2.3	2.1	2.0	1.9	1.7	1.6	1.4	1.3	1.1	0.9	0.8					
14000	2.4	2.9	3.0	2.9	2.9	2.8	2.7	2.6	2.5	2.3	2.2	2.1	1.9	1.8	1.6	1.5	1.3	1.1	0.9	0.8						
12000	3.0	3.0	3.0	3.0	2.9	2.8	2.7	2.6	2.4	2.3	2.1	2.0	1.8	1.7	1.5	1.4	1.2	1.0	0.8							
10000	3.1	3.1	3.1	3.0	2.9	2.8	2.7	2.5	2.4	2.2	2.1	1.9	1.7	1.6	1.4	1.2	1.0									
8000	4.3	4.3	4.2	4.1	4.0	3.9	3.5	3.1	2.8	2.4	2.0	1.8	1.6	1.5	1.3	1.1										
6000	5.4	5.4	5.3	5.2	5.1	5.0	4.4	3.8	3.1	2.5	1.9	1.7	1.5	1.3	1.1											
4000	6.2	6.1	6.0	5.9	5.8	5.7	4.9	4.1	3.4	2.6	1.8	1.6	1.4	1.2												
2000	6.6	6.5	6.4	6.2	6.1	5.9	5.1	4.2	3.4	2.5	1.7	1.4	1.2													
0	6.9	6.8	6.7	6.5	6.3	6.2	5.3	4.3	3.4	2.5	1.5	1.3														

b

FIGURE 20 - MOCK A320 – 1-gee accel (ΔKTAS/sec at full-power in level flight). a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

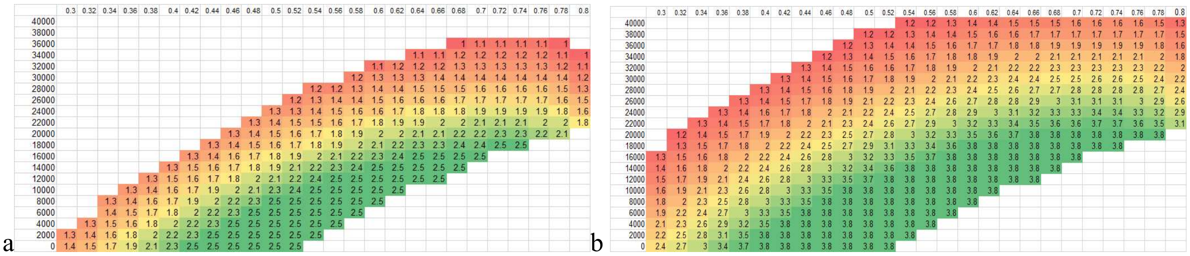

FIGURE 21 - MOCK A320 – Instant Turn Envelope – *nZ (M,ALT)* in gees. a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

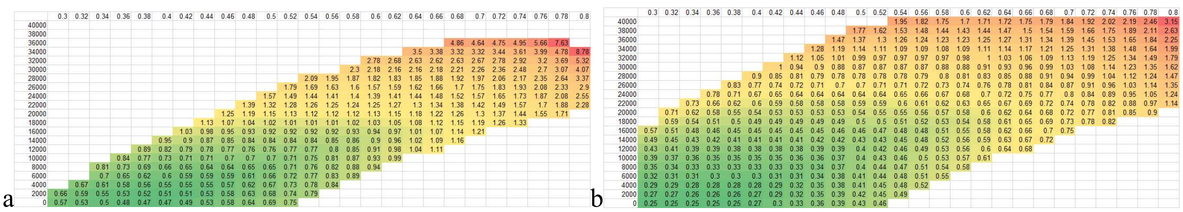

FIGURE 22 - MOCK A320 – Instant Turn Radius (M,ALT) in nM. a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

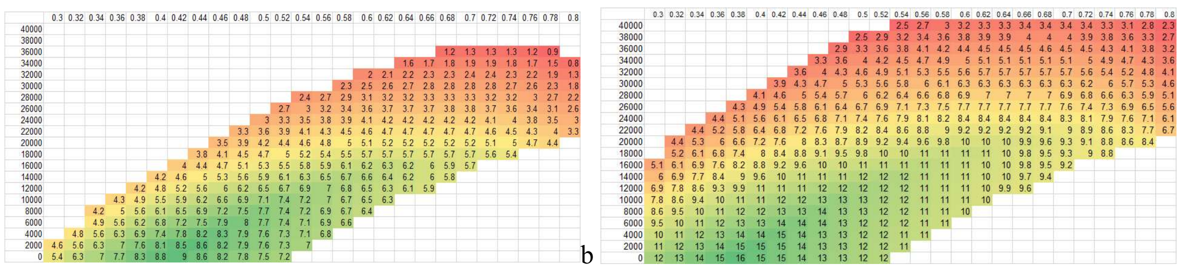

FIGURE 23 - MOCK A320 – Instantaneous Heading Change(M,ALT) – compass-degrees/sec a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

FIGURE 21 shows the maximum instantaneous turn envelope of this aircraft at two flight weights as limited by *nZmax* and MD=0.82 and *Vd*=350-KEAS. From sea-level through 16,000-ft at heavy weights, the aircraft can exceed its *VA* speed (speed at which >2.5-gee for FIGURE 21a, >3.8-gee for FIGURE 21b); in other words, its maximum instantaneous turn *Nz* is limited by structures. Above these transition altitudes, the *VA* speed is outside of the aerodynamic and propulsive limits to the steady-state flight envelope.

FIGURE 22 shows the maximum instantaneous turn radius (in nM) of this aircraft at two flight weights as limited by *nZmax* and *MD*=0.82 and *VD*=350-KEAS. At heavy weights, at sea level, the optimum speed for a minimum turn radius of 0.47-nM is at *M*=0.4 (i.e. 265-KEAS). At *W*=100,000-lbm, the instantaneous turn radius shrinks to only 0.25-nM with an optimum speed at sea-level of *M*=0.36 (~240-KEAS).

FIGURE 23 shows the maximum instantaneous heading rate change in terms of compass-degrees/sec. At heavy weights, at low altitudes the aircraft can change heading at up to 9-deg/sec; at *W*=100,000-lbm, the rate increases to 16-deg/sec. Peak instantaneous heading rate change declines sharply with altitude.

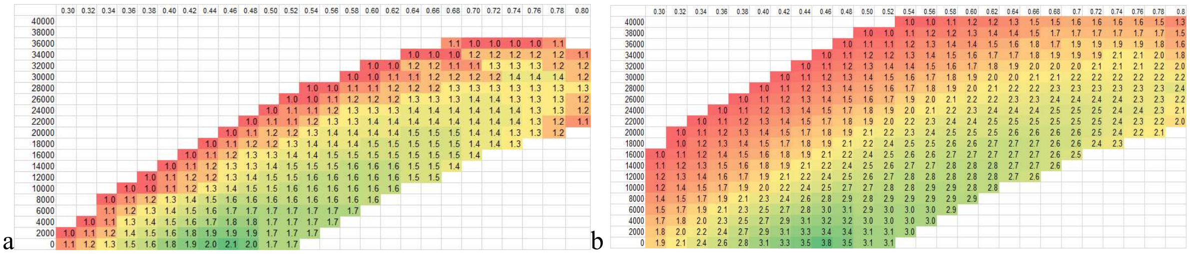

FIGURE 24 - MOCK A320 – Sustained Turn Envelope – *nZ(M,ALT)* in gees. a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

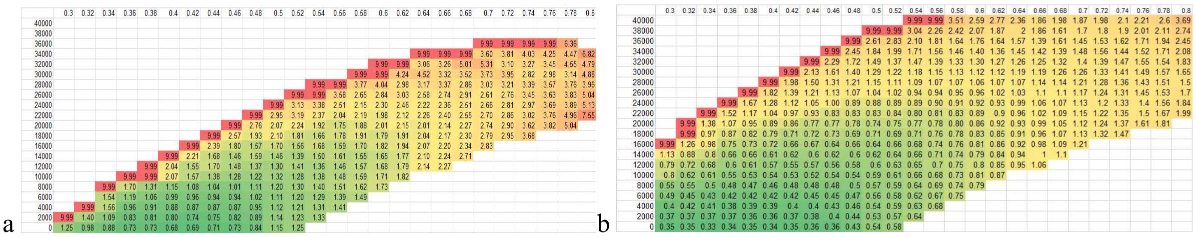

FIGURE 25 - MOCK A320 – Sustained Turn Radius (M,ALT) in nM without loss of speed or altitude. a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

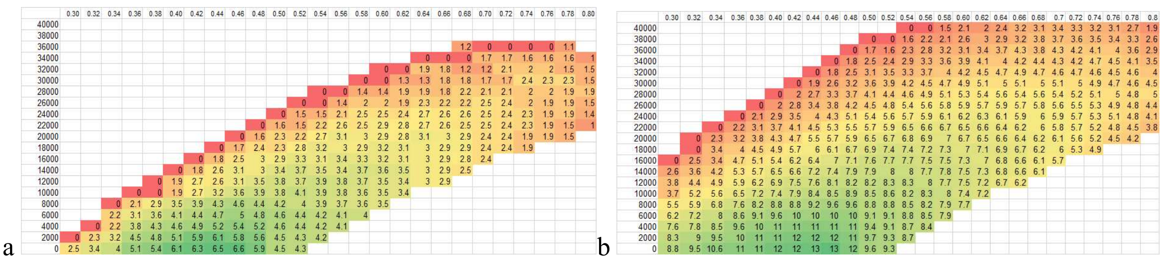

FIGURE 26 - MOCK A320 – Sustained Heading Change (M,ALT) – compass degrees/sec without loss of speed or altitude. a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

FIGURE 24 shows the maximum sustained turn capability attainable without loss of speed or altitude. At heavy weights, it is limited to ~2.1-gees at sea-level *M*=0.48 (315 KEAS). At *W*=100,000-lbm, the gee capability increases to ~3.8-gees (basically just touching on the *VA* speed) at sea-level *M*=0.46 (305 KEAS).

FIGURE 25 shows the sustained turn radius in nM at the maximum *nZ* attainable without loss of speed or altitude. At heavy weights, it is ~0.68-nM at sea-level *M*=0.4 (265 KEAS). At *W*=100,000-lbm, the sustained turn radius tightens to ~0.33-nM at sea-level *M*=0.34 (225 KEAS).

FIGURE 26 shows the sustained heading-rate-change at the maximum *nZ* attainable without loss of speed or altitude. At heavy weights, it is ~6.6-deg/sec sea-level *M*=0.46 (305 KEAS). At *W*=100,000-lbm, the heading rate change capability improves to ~13-deg/sec at sea-level also at *M*=0.46 (305 KEAS).

For comparison, the McDonnell F-4E Phantom had ~14-deg/sec peak sustained heading-rate-change capability at sea-level and 42,777-lbm – and then only at the much faster flight speed of M~0.8; refer to FIGURE 27, overleaf. [62] At M~0.4 near sea-level, a light weight A320 has comparable sustained turn heading rate change capability to the Phantom II; very impressive. At FL300, the light weight A320 can still sustain 5-deg/sec heading rate change just like the Phantom II. Thus, across a wide range of altitude (and a narrow range of speeds) the A320 and the F-4E have similar sustained gee turn capabilities. Seeing that both the US Air Force Thunderbirds and the US Navy Blue Angels flight demonstration team flew the F-4 as an airshow demonstration aircraft, we can see how a light weight narrow body airliner can produce a similarly impressive airshow display in terms of maneuverability and agility.

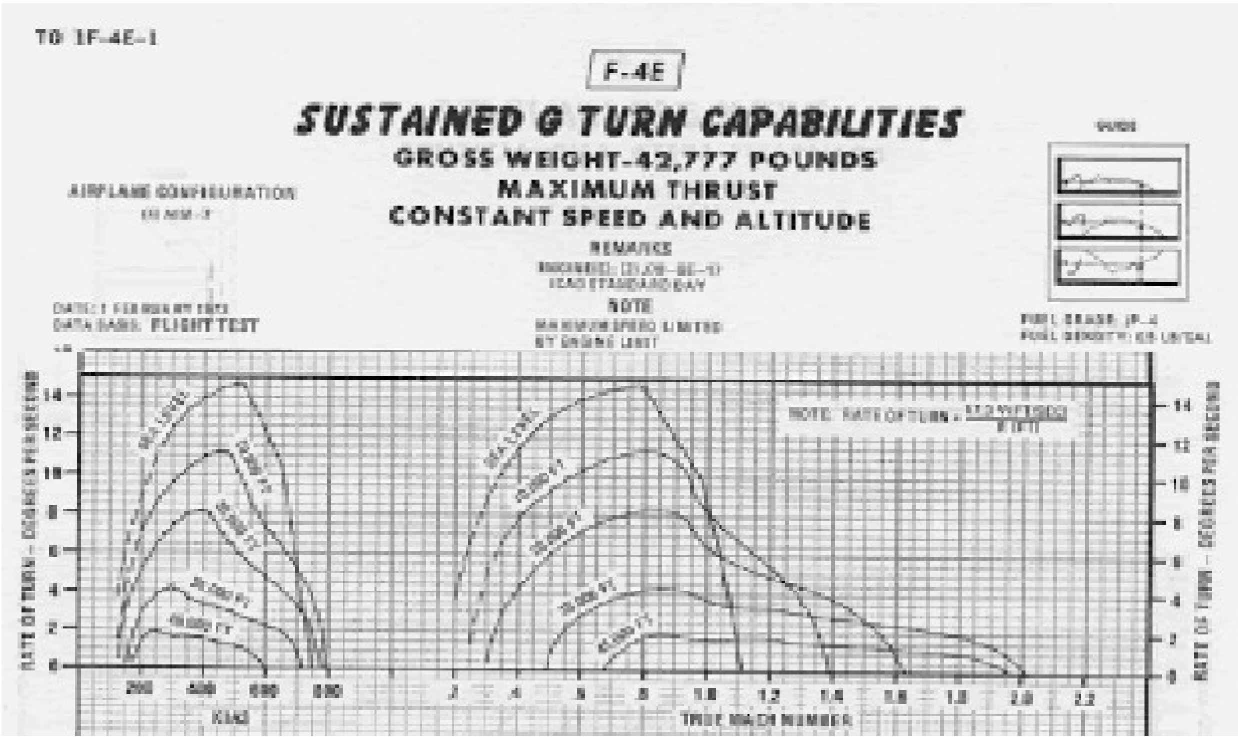

FIGURE 27 – excerpt from F-4E Phantom II Flight Manual [62]

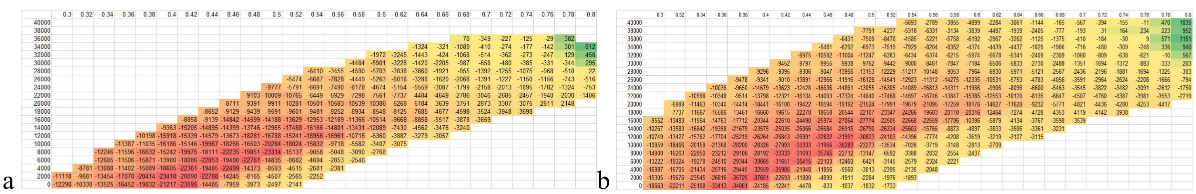
FIGURE 28 - MOCK A320- Unaccelerated Rate-of-Climb at *CLmax* (ft/min). a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

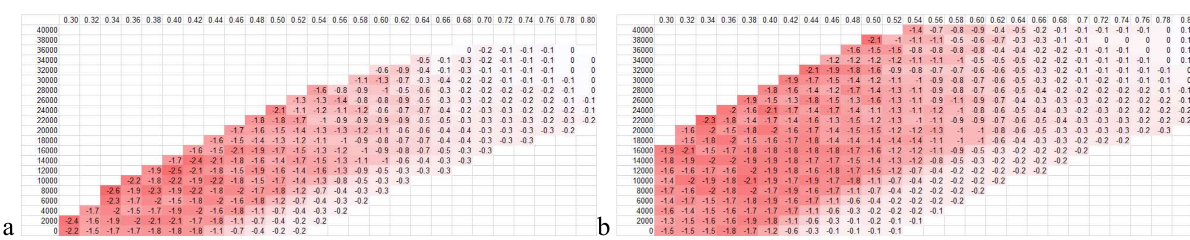
FIGURE 29 - MOCK A320 – Front Side Acceleration Potential (ΔKTAS/deg heading change). a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

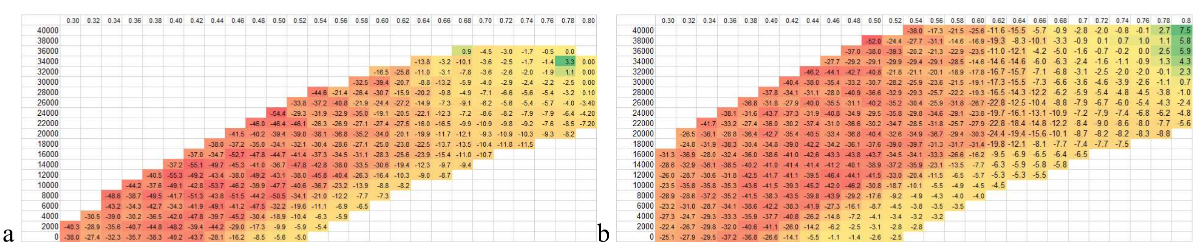
FIGURE 30 - MOCK A320 – Back Side Unaccelerated-Rate-of-Climb (Δft/deg heading change). a) *MTOW*=175,000-lbm, b) *W*=100,000-lbm

Finally FIGUREs 28, 29 and 30 (prior page) shows the unaccelerated rate of climb, the front-side acceleration potential at constant altitude and the back-side unaccelerated rate-of-climb associated with flight at maximum instantaneous turn capability. Basically these three charts give the pilot an idea as to how much speed or altitude loss is associated with a maximum *Nz* turn.

Let us consider flight beginning at M=0.70 and FL300. Across all weights, we see (refer to FIGURE 29) that the aircraft will decelerate to the tune of 0.1-KTAS per degree heading change. Thus, a violent 45-degree heading change flown above the sustained-gee limit will only lead to the loss of ~4.5-KTAS (a loss of less than 0.01 Mach number) over the course of the turn. Alternatively, if we were to hold speed and allow for a loss in altitude (~4-ft per degree of heading change) even a 180-degree heading change would result in the loss of less than 1,000-ft in altitude. Note that at heavy weights, the aircraft has ~2.8-deg/sec heading rate change capability; at 100,000-lbm it has ~6.3-deg/sec heading rate change capability. Once again, a lightly loaded narrow body transport demonstrates maneuverability quite comparable to the Phantom.

VI. Summary & Conclusions

This paper presents additional mathematics to estimate the maneuverability and agility of aircraft. While the speed / altitude / weight variation in performance is the result of highly coupled relationships balancing lift against weight and thrust against drag, it is straightforward to implement in code.

Looking at the estimated performance of a typical transport category aircraft, something reminiscent of an Airbus A320, we can see wide variations in the predicted maneuverability of changes with speed and altitude; for example, the best speed for heading-rate-change is often quite different from the best speed for minimum turning circle.

We also see here that while “sustained turn” maneuverability requirements are a classic metric, if we operate an airframe beyond its “sustained turn” capability we may see a substantial increase in agility with only a nominal loss of speed and/or altitude.

The usable maneuvering capability of a modern transport-category aircraft, when flown at light weight, can match last-generation combat aircraft. It should be no surprise then, the magnificent airshow routines flown by transport-category aircraft at Paris, Farnborough and the RIAT.

References

1. Demerly, T., “Because I was inverted! (AGAIN!)” See: https://theaviationist.com/2018/07/19/because-i-was-inverted-again-c-130-super-hercules-pilot-gets-upside-down-at-farnborough/ (accessed June 30,2021)
2. Anon., “Lockheed Martin LM-100J Commercial Freighter Receives FAA Type Certificate,” See: https://news.lockheedmartin.com/2019-11-18-Lockheed-Martin-LM-100J-Commercial-Freighter-Receives-FAA-Type-Certificate-Update#:~:text=This%20particular%20FAA%20certification%20allows%20the%20LM-100J%20to,variant%20of%20the%20military%20proven%20C-130J%20Super%20Hercules. (accessed June 30,2021)
3. Department of Commerce Aeronautics Bulletin 7-A (1933)
4. 14 CFR § 04 et seq (1938)
5. 14 CFR § 23 et seq (2021)
6. 14 CFR § 25 et seq (2021)
7. 14 CFR § 04.119 (1938)
8. 14 CFR § 04.120 (1938)
9. 14 CFR § 04.121 (1938)
10. 14 CFR § 04.210 (1938)
11. 14 CFR § 04.213 (1938)
12. 14 CFR § 04.111 (1938)
13. 14 CFR § 04.112 (1938)
14. 14 CFR § 04.113 (1938)
15. 14 CFR § 04.115 (1938)

16. 14 CFR § 04.116 (1938)
17. 14 CFR § 04 Table 04-01 (1938)
18. 14 CFR § 04 Table 04-03 (1938)
19. 14 CFR § 04 Table 04-04 (1938)
20. 14 CFR § 04 Table 04-05 (1938)
21. 14 CFR § 04.2131 (1938)
22. 14 CFR § 04.2132 (1938)
23. 14 CFR § 04.2133 (1938)
24. 14 CFR § 04.2134 (1938)
25. 14 CFR § 04.2135 (1938)
26. 14 CFR § 04 FIG 04-03 (1938)
27. 14 CFR § 04.2150 (1938)
28. 14 CFR § 04.2211 (1938)
29. 14 CFR § 04.2220 (1938)
30. 14 CFR § 04.217 (1938)
31. 5 FR 1835 (1940)
32. 14 CFR § 04.2211 (1940)
33. 10 FR 1925 (1945)
34. 14 CFR § 04.200 (1945)
35. 14 CFR § 04.201 (1945)
36. 14 CFR § 04.202 (1945)
37. 14 CFR § 04.203 (1945)
38. 14 CFR § 04.210 (1945)
39. 14 CFR § 04.2130 (1945)
40. 14 CFR § 04.2111 (1945)
41. 14 CFR § 04.2110 (1945)
42. 14 CFR § 25.301 (2021)
43. 14 CFR § 25.302 (2021)
44. 14 CFR § 25.337 (2021)
45. See: https://www.aopa.org/advocacy/advocacy-briefs/understanding-part-23-rewrite (accessed June 30,2021)
46. 14 CFR § 23.2230 (2021)
47. 14 CFR § 23.2200 (2021)
48. 14 CFR § 23.2215 (2021)
49. 14 CFR § 25.25 (2021)
50. 14 CFR § 25.143 (2021)
51. 14 CFR § 25.147 (2021)
52. 14 CFR § 25.253 (2021)
53. 14 CFR § 25.149 (2021)
54.. Takahashi, T.T., *Aircraft Performance & Sizing, Vol. I: Fundamentals of Aircraft Performance*, Momentum Press, New York, NY, 2016.
55.Gallagher, G.L., Higgins, L.B., Khinoo, L.A. and Pierce, P.W., “Fixed Wing Performance,” U.S. Naval Test Pilot School Flight Test Manual, USNTPS-FTM-NO.108, 30 Sept. 1992.
56. Feagin, R.C. and Morrison, W.D. “Delta Method, An Empirical Drag Buildup Technique,” NASA CR 151971, December 1978.
57. NPSS, Numerical Propulsion System Simulation, Software Package, Ver. 2.8, Ohio Aerospace Institute,
58. Beard, J.E. and Takahashi, T.T., “Optimal Piloting Approaches For Obstacle Clearance Limited Standard Instrument Departures,” AIAA 2018-0285, 2018.
59. Wilson, J. and Takahashi, T.T., “The Doghouse Plot: History, Construction Techniques and Application,” AIAA 2017-3266, 2017.
60. A320 AFM
61. Coram, J. *Boyd: The Fighter Pilot Who Changed the Art of War*, Back Bay Books, 2004.
62. 1F-4E-1 AFM