LONGITUDINAL CONTROL LAWS BASED ON C* CRITERION

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Abstract. *The aim of this work was the development of longitudinal control laws based on the C* criterion of handling qualities. To achieve this objective a 50 passengers regional jet model was used. Simulations were performed with the aircraft model on its open-loop and closed-loop configurations. The closed-loop configuration showed a better dynamic response, satisfying the C* criterion, while the open-loop configuration showed an oscillatory response with little damping and did not satisfy the C* criterion. The system was also evaluated by other flying qualities criteria, like the military specifications and the Gibson criterion. In addition, simulations with the closed-loop configuration were performed varying the aircraft altitude, speed and mass to observe their influences on the aircraft behavior. The system robustness was also analyzed by varying the aircraft stability derivatives. The designed control system showed to be robust in order of being able to adapt to various flight conditions, turning these parameters not able to modify significantly the aircraft response.*

Keywords*: C* criterion, handling quality, stability augmentation system*

1. INTRODUCTION

Modern commercial aircrafts are designed to reduce as much as possible their operating cost. In order to reduce this cost, drag should be minimized, which can be done by reducing the aircraft tail volume. As a consequence, the stability is decreased or even eliminated.

To pilot the aircraft, a control system is required in order to stabilize it. This control system is the interface between the pilot commands and the aircraft control surfaces. Nowadays, commercial aircrafts are designed with a reduced tail volume, i.e., they have a relaxed stability. Therefore fly-by-wire systems are developed to turn the aircraft stable.

The purpose of this work was the development of longitudinal control laws based on the C* criterion of handling qualities.

2. HANDLING-QUALITIES

2.1. Handling-qualities requirements

Control-law design can only be performed satisfactorily if a set of design requirements or performance criteria is available. In case of control systems for piloted aircraft, generally applicable quantitative design criteria are very difficult to obtain. The reason for this is that the ultimate evaluation of a human-operator control system is necessarily subjective and, with aircraft, the pilot evaluates the aircraft in different ways depending on the type of aircraft and phase of flight (Stevens and Lewis, 2003).

The Cooper-Harper scale is a systematic approach to handling-qualities evaluation through pilot opinion rating (Fig. 1). Once a rating scale like this has been established it is possible to begin correlating the pilot opinion rating with the properties of the aircraft dynamic model, and hence derive some analytical specifications that will guarantee good handling qualities. Although this may seem simple in principle, it has proven remarkably difficult to achieve in practice, and after many years of handling-qualities research it is still not possible to precisely specify design criteria for control systems intended to modify the aircraft dynamics (Stevens and Lewis, 2003).

It will be considered first some possible ways in which requirements for dynamic response may be specified. The aircraft model may be linearized in a particular flight condition and the poles and zeros, or frequency response, of a particular transfer function compared with a specification. Alternatively, certain time responses may be derived from the nonlinear model, in a particular flight condition, and be compared with specifications (Stevens and Lewis, 2003).

Table 1. Pilot opinion rating and flying qualities level (Stevens and Lewis, 2003)

Aircraft Characteristics	Demands on Pilot in Selected Task or Required Operation	Pilot Rating	Flying Qualities Level
Excellent; highly desirable	Pilot compensation not a factor for desired performance	1	1
Good; negligible deficiencies	as above	2	
Fair; some mildly unpleasant deficiencies	Minimal pilot compensation required for desired performance	3	
Minor but annoying deficiencies	Desired performance requires moderate pilot compensation	4	2
Moderately objectionable deficiencies	Adequate performance requires considerable pilot compensation	5	
Very objectionable but tolerable deficiencies	Adequate performance requires extensive pilot compensation	6	
Major deficiencies	Adequate performance not attainable with maximum tolerable pilot compensation Controllability not in question	7	3
Major deficiencies	Considerable pilot compensation required for control	8	
Major deficiencies	Intense pilot compensation required for control	9	
Major deficiencies	Control will be lost during some portion of required operation	10	

2.2. The military flying-qualities specifications

The U.S. Military Specification for the Flying Qualities of Piloted Airplanes (MIL-F-8785C, 1980) does provide some analytical specifications that must be met by U.S. military aircraft (Stevens and Lewis, 2003).

The military specification defines airplane classes, flight phases, and flying qualities levels, so that different modes can be specified for the various combinations (Tab. 2). The flying qualities levels are linked to the Cooper-Harper ratings as shown in Tab. 1 (Stevens and Lewis, 2003).

2.2.1. Phugoid specifications

The military specification dictates that for the different levels of flying qualities, the damping ζ_p and natural frequency ω_{np} of the Phugoid mode will satisfy the following requirements (Stevens and Lewis, 2003):

Level 1: $\zeta_p \geq 0.04$
Level 2: $\zeta_p \geq 0.0$
Level 3: $T_{2p} \geq 55.0$ s

In the level-3 requirement the mode is assumed to be unstable, and T_{2p} denotes the time required for the mode to double in amplitude (Stevens and Lewis, 2003).

Table 2. Definitions – flying qualities specifications (Stevens and Lewis, 2003)

Airplane Classes	
Class I	Small, light airplanes.
Class II	Medium weight, low-to-medium-maneuverability airplanes.
Class III	Large, heavy, low-to-medium-maneuverability airplanes.
Class IV	High-maneuverability airplanes
Flight Phases	
Category A	Nonterminal flight phases generally requiring rapid maneuvering.
Category B	Nonterminal flight phases normally accomplished using gradual maneuvers without precision tracking, although accurate flight-path control may be required.
Category C	Terminal flight phases normally accomplished using gradual maneuvers and usually requiring accurate flight-path control.
Flying Qualities Levels	
Level 1	Flying qualities adequate for the mission flight phase.
Level 2	Flying qualities adequate to accomplish the mission flight phase, but some increase in pilot workload or degradation in mission effectiveness exists.
Level 3	Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive, or mission effectiveness is inadequate, or both.

2.2.2. Short-period specifications

The short-period requirements are specified in terms of the natural frequency and damping of the "short period mode" of the equivalent low-order system. Table 3 shows the requirements on the equivalent short-period damping ratio ζ_{sp} (Stevens and Lewis, 2003).

Table 3. Short-period damping ratio limits (Stevens and Lewis, 2003)

	Cat. A & C Flight Phases		Cat. B Flight Phases	
Level	Minimum	Maximum	Minimum	Maximum
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15*	no limit	0.15*	no limit

* May be reduced at altitude > 6096 m with approval.

The requirements on equivalent undamped natural frequency (ω_{nsp}) are given in Tab. 4 and are specified indirectly, in terms of the quantity $\omega_{nsp}^2 / (n/\alpha)$. The denominator ($n/\alpha$) of this term is the aircraft load-factor response to angle of attack in g's per radian, which can also be given by:

$$\frac{n}{\alpha} = \frac{V}{g \cdot T_{\theta 2}} \tag{1}$$

where V is the aircraft speed and $T_{\theta 2}$ is a time constant associated with the elevator-to-pitch-rate transfer function zero (Steven and Lewis, 2003).

The term $\omega_{nsp}^2 / (n/\alpha)$ is known as Control Anticipation Parameter, CAP (Field, 1993).

Table 4. Limits on $\omega_{nsp}^2/(n/\alpha)$ (Stevens and Lewis, 2003)

	Cat. A Phases		Cat. B Phases		Cat. C Phases	
Level	Min.	Max.	Min.	Max.	Min.	Max.
1	0.28 $\omega_n \geq 0.1$	3.60	0.085	3.60	0.16 $\omega_n \geq 0.7$	3.60
2	0.16 $\omega_n \geq 0.6$	10.00	0.038	10.0	0.096 $\omega_n \geq 0.4$	10.00
3	0.16	no limit	0.038	no limit	0.096	no limit

There are some additional limits on the minimum value of n/α and the minimum value of ω_n, for different classes of airplane in category C.

2.3. C* criterion

The C* criterion was one of the first handling qualities criteria designed to take account of advanced aerodynamic designs of modern aircraft and higher order systems introduced by flight control systems. Several aircraft have since employed control laws based around the C* parameter. A proportional feedback C* controller was applied to a Boeing 747-100 in landing approach configuration, and assessed against the C* criterion and the US military specification MIL-STD-1797A (Field, 1993).

Pilots respond to a blend of pitch rate and normal acceleration, with the ratio varying according to natural variations in the aircraft's response. At low velocities normal acceleration cues are weak, therefore the predominant cue would be pitch rate. At high velocities where slight pitching may produce large normal acceleration changes, normal acceleration cues dominate. This blend of normal acceleration and pitch rate was named C* and is defined as (Field, 1993):

$$C^* = K_{nz} n_z + K_q q \tag{2}$$

where n_z is the normal acceleration at the pilot's station and q is the pitch rate. The dimensionless C^* parameter can be obtained by blending the outputs of a pitch rate gyro and a linear accelerometer at the pilot's station. The outputs of the two sensors would be blended with a fixed ratio, however the relative contribution of n_z and q would automatically vary with the velocity as a result of the variation inherent in the n_z and q transfer functions (Field, 1993).

The ratio of the constants K_{nz} and K_q was determined at the velocity where both cues command equal pilot attention, which was chosen as 122 m/s. The ratio is then given by (Field, 1993):

$$\frac{K_q}{K_{nz}} = 12.4 \tag{3}$$

Figure 1 shows the C* time history envelopes.

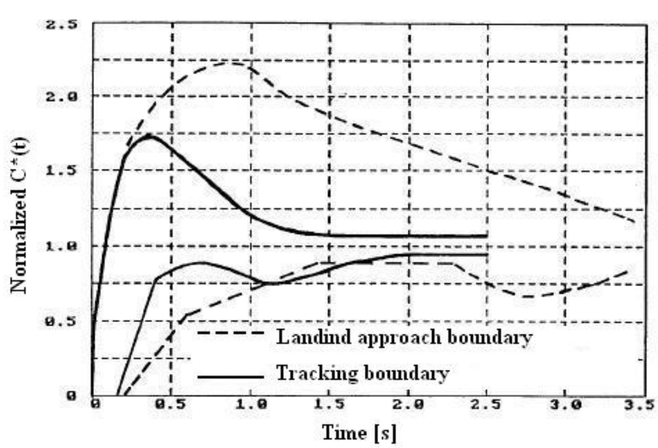

Figure 1. C* time history envelopes (Field, 1993)

The C* parameter was based on the belief that at low velocities the pilot reacts to pitch changes while at high velocities normal acceleration cues dominate. Accepting this approaching it can be proposed that at low velocities a pilot controls the flight path of his aircraft through control of the pitch attitude (and therefore pith rate) while at higher velocities he controls the flight path through control of the normal acceleration, and hence angle of attack. Thus using a C* demand control system he is able to directly control these parameters throughout the speed range of the aircraft (Field, 1993).

3. MATHEMATICAL MODELING

3.1 Aircraft modeling

The aircraft longitudinal equations (decoupled) used in this work are presented bellow:

$$\begin{aligned}
\dot{V} &= \frac{F\cos(\alpha+\alpha_F) - \frac{1}{2}\rho V^2 S C_D - mg\sin\gamma}{m} \\
\dot{\gamma} &= \frac{F\sin(\alpha+\alpha_F) + \frac{1}{2}\rho V^2 S C_L - mg\cos\gamma}{mV} \\
\dot{\alpha} &= q - \dot{\gamma} \\
\dot{q} &= \frac{\frac{1}{2}\rho V^2 S l C_m + F\cos\alpha_F \cdot z_F}{I_y} \\
\dot{H} &= V\sin\gamma
\end{aligned} \tag{4}$$

where V is the aircraft true air speed, F is the thrust force, α is the angle of attack, α_F is the angle between the thrust force and the aircraft longitudinal axis, ρ is the air density, S is the wing area, m is the aircraft mass, g is the gravitational acceleration, γ is the flight path angle, q is the pitch rate, C_D is the drag coefficient, C_L is the lift coefficient, C_m is the pitch moment coefficient, l is the mean aerodynamic cord, z_F is the distance between the aircraft longitudinal axis and the thrust force axis, I_y is the inertial moment of the wing axis and H is the altitude (Etkin and Reid, 1996).

The normal acceleration at pilot's station is given by the normal acceleration at the aircraft CG (Center of Gravity) plus the pilot's station acceleration in regard to the aircraft CG:

$$a_{np} = \frac{V\dot{\gamma}}{g} + \frac{x_a\dot{q}}{g} \tag{5}$$

where a_{np} is the normal acceleration at pilot's station and x_a is distance between the pilot's station and the aircraft CG.

The C* parameter is given by:

$$C^* = 12.4q + a_{np} \tag{6}$$

3.2 Flight control system

The flight control system is presented in Fig. 2.

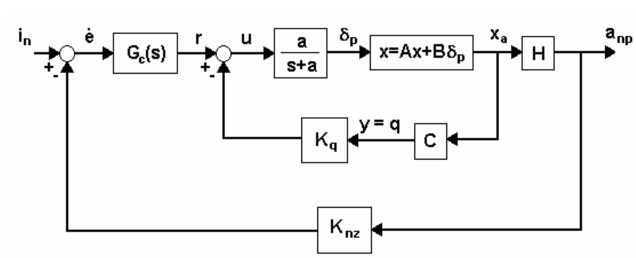

Figure 2. Flight control system

This system consists of a SAS (Stability Augmentation System) with pith rate feedback (K_q gain) closing the loop around the aircraft dynamics and the elevator actuator, a CAS (Control Augmentation System) with a PI (Proportional + Integral) controller (K_p and K_i gains), $G_c(s)$, and normal acceleration (at pilot's station) feedback (K_{nz} gain). The system input is the stick pilot command. On the elevator actuator block, the constant a represents the inverse of the time constant. On the aircraft dynamics block, **A** and **B** are the matrices of the aircraft linearized model.

The equations for the linearized model are:

$$\begin{bmatrix} \dot{x}_a \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_a - B_a\left(K_p K_{nz} H + K_q C\right) & B_a K_i \\ -K_{nz} H & 0 \end{bmatrix} \begin{bmatrix} x_a \\ e \end{bmatrix} + \begin{bmatrix} B_a K_p \\ 1 \end{bmatrix} i_n \tag{7}$$

where

$$x_a = \begin{bmatrix} V & \gamma & \alpha & q & H & \delta_p \end{bmatrix}^T \tag{8}$$

and

$$\begin{aligned} A_a &= \begin{bmatrix} A & B \\ 0 & -a \end{bmatrix} \\ B_a &= \begin{bmatrix} 0 \\ a \end{bmatrix} \\ H &= \begin{bmatrix} \frac{V_e}{g} A_a(2,:) + \frac{x_a}{g} A_a(4,:) \end{bmatrix} \\ C &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{aligned} \tag{9}$$

4. RESULTS

The model was simulated on Matlab 6.5/Simulink. The controller, pitch rate and normal acceleration at pilot's station gains were evaluated by quadratic minimization (Matlab *fmincon* function). The function used on minimization was the area between the step response and the normalized C* response, considering the C* boundary as the constraints.

The elevator actuator is of first order with time constant of 1/20 s and unit gain. Besides, the elevator minimum deflection is 0,698 rad (40°) on both directions, while the maximum deflection variation is 1,05 rad/s (60°/s) on both directions.

4.1 Open-loop response

The following matrixes were obtained for the open-loop configuration:

$$A = \begin{bmatrix} -5{,}7770e-3 & -9{,}8066 & -4{,}3517 & -1{,}0247e-1 & -1{,}8553e-5 \\ 4{,}3410e-4 & 0 & 5{,}0419e-1 & 1{,}1826e-2 & -6{,}0643e-6 \\ -4{,}3410e-4 & 0 & -5{,}0419e-1 & 9{,}8817e-1 & 6{,}0643e-6 \\ -1{,}3098e-4 & 0 & -5{,}5698 & -1{,}2266 & -5{,}6257e-6 \\ 0 & 2{,}3598e2 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 8{,}4576e-1 & -2{,}2025e-1 \\ 4{,}0200e-5 & 2{,}5419e-2 \\ -4{,}0200e-5 & -2{,}5419e-2 \\ -2{,}0624e-2 & -3{,}5187 \\ 0 & 0 \end{bmatrix}$$

Table 5 shows the short-period and phugoid parameters of the open-loop model, while Fig. 3 shows the aircraft response for a unit step input (normalized pitch rate, q, and normalized normal acceleration at pilot's station, n_{zp}).

Table 5. Short-period and phugoid parameters of the open-loop model

	Short-period mode	Phugoid mode
ω_n (rad/s)	2.4746	0.0650
ζ	0.3500	0.0201
Period (s)	2.71	96.74

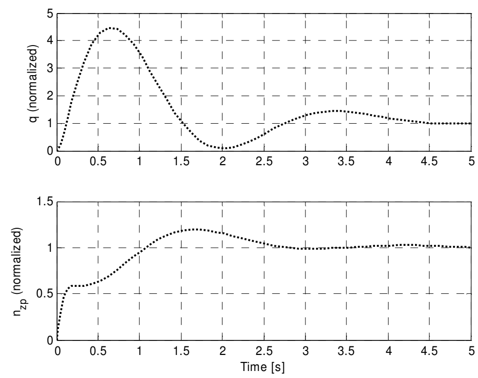

Figure 3. Aircraft response for a unit step input (open-loop configuration)

4.2 Closed-loop response

The closed-loop gains determined by optimization are:

K_p = 2,0054·10^{-1}
K_i = 1,5808·10^{-3}
K_q = -3,8705
K_{nz} = -1,0777

The matrix of the closed-loop model is the following:

$$A = \begin{bmatrix} -5{,}7770e-3 & -9{,}8066 & -4{,}3517 & -0{,}10247 & -1{,}8553e-5 & -0{,}22025 & 0 \\ 4{,}3410e-4 & 0 & 0{,}50419 & 0{,}011826 & -6{,}0643e-6 & 0{,}025419 & 0 \\ -4{,}3410e-4 & 0 & -0{,}50419 & 0{,}98817 & 6{,}0643e-6 & -0{,}025419 & 0 \\ -1{,}3098e-3 & 0 & -5{,}5698 & -1{,}2266 & -5{,}6257e-6 & -3{,}5187 & 0 \\ 0 & 235{,}98 & 0 & 0 & 0 & 0 & 0 \\ 3{,}8023e-2 & 0 & 20{,}733 & 72{,}328 & -6{,}6962e-4 & -37{,}893 & 0{,}031932 \\ 9{,}3863e-3 & 0 & 5{,}1180 & -1{,}4456 & -1{,}6530e-4 & -4{,}3677 & 0 \end{bmatrix}$$

Table 6 shows the short-period and phugoid parameters of the open-loop model, while Fig. 4 shows the aircraft response for a unit step input.

Table 6. Short-period and phugoid parameters of the open-loop model

	Short-period mode	Phugoid mode
ω_n (rad/s)	3.8291	0.0352
ζ	1.4163	0.1027
Period (s)	-	179.54

Analysis of Fig.3 and Fig. 4 shows the closed-loop configuration presented faster response, shorter stabilization time, less overshoot and less oscillation, thus presenting a better response than the open-loop configuration.

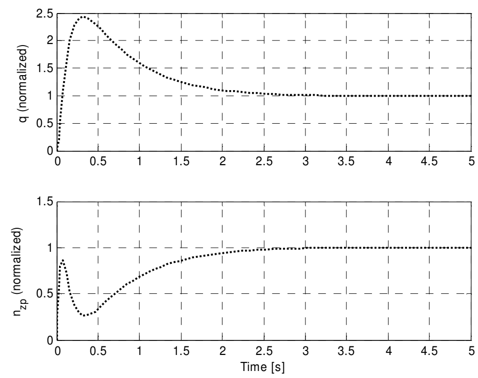

Figure 4. Aircraft response for a unit step input (closed-loop configuration)

4.3 Flying qualities criteria analysis

4.3.1 Military flying-qualities criterion

Figure 5 shows the flying-quality level for the short-period mode.

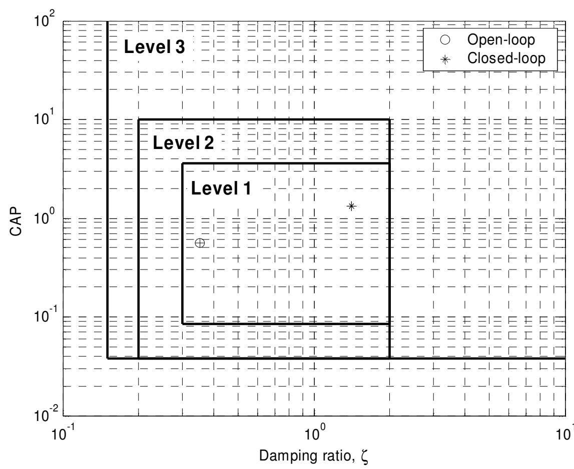

Figure 5. Flying-quality level (short-period mode)

According to Fig. 5 and Tab. 5 and 6, the open-loop configuration is classified (category B flight phase) as level 1 for short-period mode and level 2 for phugoid model, while the closed-loop mode configuration is classified as level 1 for both modes.

4.3.2 Gibson criterion

Table 7 shows the results according to Gibson Dropback criterion.

Table 7. Gibson Dropback criterion

	q_m [rad/s]	q_s [rad/s]	q_m/q_s	DB/q_s [s]
Open-loop configuration	$1.8541 \cdot 10^{-2}$	$4.1382 \cdot 10^{-3}$	4.4806	2.6295
Closed-loop configuration	$6.9988 \cdot 10^{-4}$	$2.8449 \cdot 10^{-4}$	2.4552	1.2311

According to Tab. 7, both configurations did not present good flying quality, though the closed-loop configuration parameters remained in a better flying quality region than the open-loop configuration.

The open-loop configuration was classified as optimum, with no PIO (Pilot Induced Oscillation), according to Gibson Phase Rate criterion. This criterion was not applicable to the closed-loop configuration (for detailed analysis see Paula, 2006).

4.3.3 C* criterion

Figure 6 shows the normalized C* response for the open-loop and closed-loop configurations.

As shown in Fig. 6, the C* response of the open-loop model is outside of the C* boundaries, hence indicating this configuration does not comply with the C* criterion. However, the C* response for the closed-loop model remained inside the C* boundary, thus indicating this configuration complies with the C* criterion.

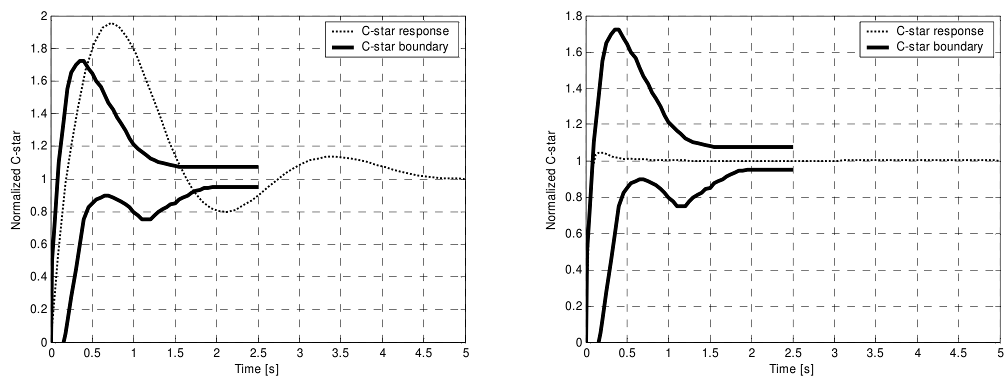

Figure 6. Normalized C* response for the open-loop (left) and closed-loop (right) configurations

4.4 Parametric analysis

The aircraft behavior was simulated in different flying conditions, considering variations in altitude, speed and mass, to observe the influence of these parameters in the aircraft response. Each parameter was modified considering the others constants. The closed-loop gains were calculated again for all the conditions mentioned above.

The altitude was varied from its maximum operating altitude, 12,497 m (41,000 ft), to 10,668 m (35,000 ft) in decrements of 914 m (3,000 ft). Figure 7 shows the aircraft response and the C* response for the three different altitudes.

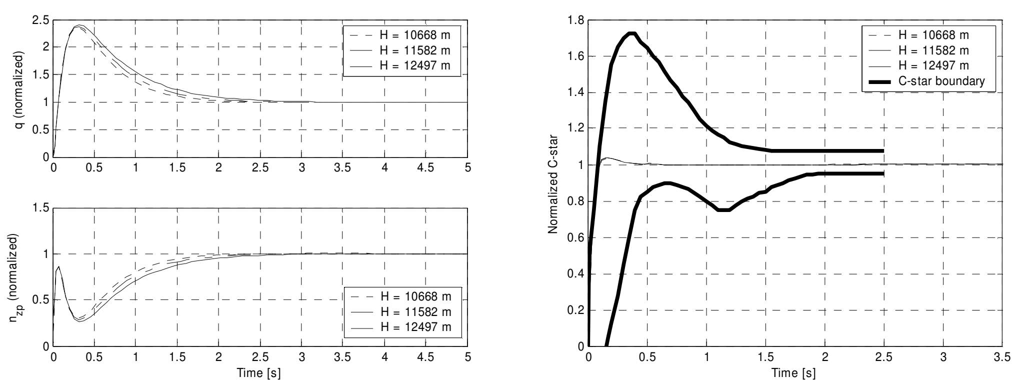

Figure 7. Aircraft and C*responses for different altitudes

Similar results were obtained varying the aircraft mass and speed. It was considered a decrease of 2,000 Kg on the aircraft mass (fuel consumption) and a decrease of 10 m/s on the aircraft speed (weather condition). The complete analysis is found in Paula, 2006.

The results show the closed-loop model can adapt to different flying conditions and presents similar behavior.

4.5 Robustness analysis

Some stability derivatives were increased in 10% to test the model robustness (the closed-loop gains were kept the same). The following stability derivatives were modified: $C_{L\alpha}$, $C_{L\delta}$, $C_{m\alpha}$, $C_{m\delta}$. Figure 8 shows the aircraft and C* responses for variation of the $C_{L\alpha}$ derivative. Similar results were obtained for variation of the other derivatives (for detailed analysis see Paula, 2006).

The results show the system is robust regarding to little variation of its stability derivatives.

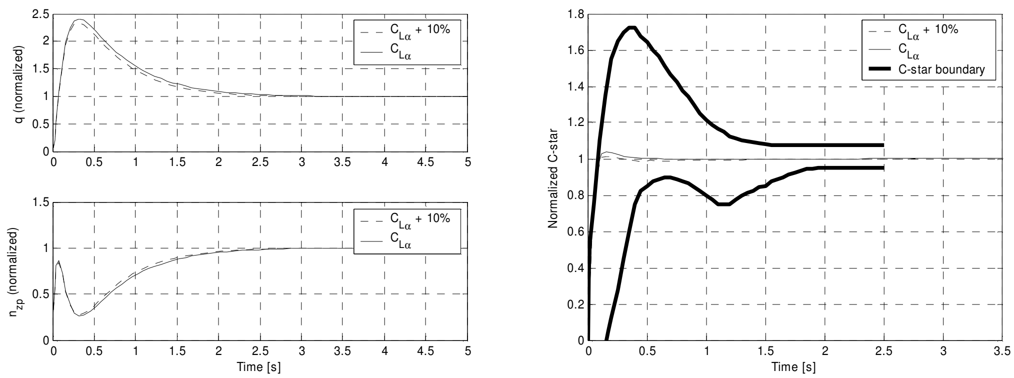

Figure 8. Aircraft and C*responses for variation of $C_{L\alpha}$ derivative

Paula (2006) describes a relaxed stability analysis, in which a stability derivative was modified to turn to level 3 the aircraft flying quality (short-period mode) for open-loop configuration.

5. CONCLUSION

This work presented a study of longitudinal aircraft control based on C* criterion of handling quality. In open-loop configuration the model did not comply with the C* criterion, presenting for military specification level 1 for short-period mode and level 2 for phugoid mode. In closed-loop configuration the model complied with the C* criterion, presenting level 1 for both modes in longitudinal control.

The parametric analysis showed the closed-loop configuration efficiency regarding to variation of aircraft altitude, speed and mass. Variation on the stability derivatives did not affect the performance of the closed-loop configuration, showing the robustness of this model.

Hence the use of C* based controllers has been shown applicable to the aircraft under study.

6. REFERENCES

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7. RESPONSIBILITY NOTICE

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